MAINTENANCE MANPOWER REALLOCATION ASSESSED BY STOCHASTIC MODELS

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THESIS

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ASSESSED BY
STOCHASTIC MODELS

by

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Three models of an aircraft repair facility are developed for use in computing manpower savings achievable by scaling-up aircraft maintenance shops. One model is a bivariate Markov process model requiring a Gauss-Seidel iterative algorithm for its solution. The other two are the simple Repairman problem and the M/M/S queue, the solutions to which can be found in most introductory texts on stochastic models. The

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simple models are compared to the more refined bivariate model in regard to predictive accuracy. The simple models are used to compute the manpower savings achievable in one scaling-up scheme, and predict a 20% savings in maintenance personnel.

Maintenance Manpower Reallocation
Assessed by
Stochastic Models

by

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B.S.E.E., Purdue University, 1967

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K	total number of shops at the repair facility.
M	total number of aircraft at the repair facility.
B	as a subscript denotes back-shop activity. Also used as a random variable indicating the number of aircraft down at a shop for back-shop repair.
F	as a subscript denotes flight-line activity. Also used as a random variable indicating the number of aircraft down at a shop for flight-line repair.
λ	arrival or failure rate
R	the set which characterizes the repair shops
S_s	the number of repairmen at the shop or shop portion denoted by the subscript s
μ	repair or service rate
P	the set of performance measures which characterize the repair facility
E(arg)	the expected value of the argument
Var(arg)	the variance of the argument
N_i	number of aircraft down for repair at the shop indicated by the subscript i
N_s	number of possible states of the bivariate model
Q_i	number of aircraft in the queue at the shop indicated by the subscript i
W_i	waiting time at the shop indicated by the subscript i
D_i	delay time at the shop indicated by the subscript i
P_{fb}	the long-run probability of exactly f aircraft down for flight-line repair and b aircraft down for back-shop repair
f	the number that the random variable F assumes

b	the number that the random variable B assumes
$P_{fb}^{(k)}$	the kth iteration value of P_{fb}
P_f	the long-run probability of f aircraft down for flight-line repair
P_b	the long-run probability of b aircraft down for back-shop repair
P_n	the long-run probability of n aircraft down for repair ($n = f + b$)
L	the expected number of aircraft in the system
λ_{Fa}	the average arrival rate at the flight-line
λ_{Ba}	the average arrival rate at the back-shop
λ_a	the average arrival rate into the shop, flight-line and back-shop both

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I. INTRODUCTION

It is quite a three-pipe problem.
-Sir Arthur Conan Doyle, The Red-headed League

A. BACKGROUND

Under the auspices of the United States Air Force Project Rand, the RAND Corporation is currently studying ways to improve the aircraft maintenance posture of the United States Air Force. In the past, RAND's Logistics Studies Program has been primarily concerned with the improvement of particular logistics functions. They have since departed from this basically micro approach to logistics problems for several reasons. First, the Air Force has over the years developed an in-house capability to analyze micro-oriented logistics problems, so that RAND assistance in this area is no longer required to the extent that it was in the past. Second, with the Air Force logistics analysts now concentrating on the micro-oriented problems, RAND's analysts are free to concentrate their efforts on macro-oriented approaches to the total logistics system. "If the Air Force can take care of the trees, then let us study the forest," is a good, although casual, expression of the present RAND approach to logistics problems. This ivory tower attitude may result in the introduction of new logistics structures in the future that will dramatically reduce support costs.

This shift in emphasis from a micro approach to logistics function, to a concentration on macro-oriented problems, is a fairly recent development. In July, 1973 a project called ANALOGS 1980s (Analysis of Logistics Structures for the 1980s) was formally initiated as a part of RAND's Logistics Studies Program. Sponsored by Lieutenant General William W. Snavely, Deputy Chief of Staff, Systems and Logistics, Headquarters, USAF, the project has as its explicit goal the exploration of logistics structures that could dramatically reduce support costs without degrading mission capability. Later, on September 9, 1974, the Deputy Chief of Staff, Systems and Logistics, provided further impetus to the macro approach when he issued a Program Management Directive authorizing a Maintenance Posture Improvement Program (MPIP) with two broad objectives:

1. To reduce costs of maintenance in peacetime and wartime contingencies.
2. To increase the effectiveness of maintenance in support of operational missions.

Information on ANALOGS 80s and on work to support the MPIP has been provided by S. Drezner and I. Cohen of RAND, to whom the author is grateful.

As a result of RAND work done in support of MPIP and the ANALOGS 80s project, there is a current interest in two organizational structures which attempt to reduce support costs by scaling-up maintenance activities, both of which

represent a departure from the organizational structures presently employed by the USAF. The older structure of the two, called Queen Bee, specifies the centralization of certain intermediate level aircraft maintenance at large central bases called Queen Bees. The second structure, called the Reallocation of Activities Alternative (RAA), grew out of the ANALOGS 80s project analysis. RAA specifies the decentralization of combat-mission-oriented activities at Combat Mission Bases (CMBs), and the centralization of logistics and training activities at Support Mission Bases (SMBs). Costs and payoffs calculations for each of the two structures are needed before a decision to implement is made regarding either structure. In late September, 1974, work began in earnest on the development of models which could be used to assess the effectiveness of such configurations.

B. MOTIVATION

At the outset of the modeling effort, it was felt that the models developed could be used in a broader context than the RAND study. The Air Force is not the only branch of the armed forces with an aircraft logistics problem; the Navy has one too. Additionally, there were similarities between the structures employed by the two branches, so that the models developed for the Air Force could be applied to Navy problems with little or no modification. There was, thus, strong motivation for a thesis focusing on the modeling of aircraft logistics structures.

C. USES

1. Analysis of Current Structure

Once models for the current structure have been developed, they could provide answers to several cogent questions. For example: Is the present manpower allocation within the current structure reasonably satisfactory, if not optimal. How might the manpower be reallocated to improve readiness? What degradation in readiness can be expected for various reductions in manpower? If manpower reductions are necessary, where will the effects of the cuts degrade readiness the least?

Most importantly, models for the current structure also provide a basis for the comparison of the current structure with a proposed alternative structure. They can also be used to analyze the effects of increasing the number of aircraft that the current maintenance structure must support. Alternatively, they can be used to indicate acceptable manpower cuts in the event that the number of aircraft and/or the amount of flying is decreased. Provided that reliability estimates of a new proposed weapon system were available, the models would be useful in predicting manpower costs in support of a new weapons system. It may be that an alternative support system may be better suited for the proposed weapon system than the current support system. The models would help resolve problems such as these.

2. Analysis of Alternative Structures

As stated previously, before decisions are made regarding alternative structures, the current system should

be modeled to provide a basis for comparison of the current structure with the proposed structure. In the RAND study, the current structure was modeled first. This provided measures of performance for the system. Then the alternative structure was modeled. The model of the alternative structure was used to show that measures of performance on par with those attainable with the current structure were achievable with the proposed structure with fewer personnel.

There is a trap to watch for in this kind of comparative analysis. If the manning of the current support system is not optimal, then the comparison with the alternative structure, assumed operating optimally, injects a bias in favor of the alternative structure into the analysis. It may be that the reallocation of manpower within the current structure will yield measures of performance that the alternative structure cannot satisfy with less personnel.

3. Application to Other Support Structures

It was stated earlier that there were similarities between Air Force and Navy aircraft support structures. There are also similarities between aircraft support structures and ship support structures. If there are economies attainable by scaling-up certain logistics functions within the aviation community, then similar economies may exist in the surface and submarine communities.

D. SCOPE

A logistics support structure has not been defined up to this point. A logistics support structure is defined as

the organization, training, quantification, and geographical positioning, where each is applicable, of military and civilian personnel, tools, equipment, armament, spare parts, fuel and physical plant facilities with the explicit purpose of maintaining a weapon system in a satisfactory state of readiness to perform peacetime missions and meet wartime or combat contingencies. This nebulous definition is provided to point out the magnitude of the task confronting the analyst who aspires to model the structure in toto.

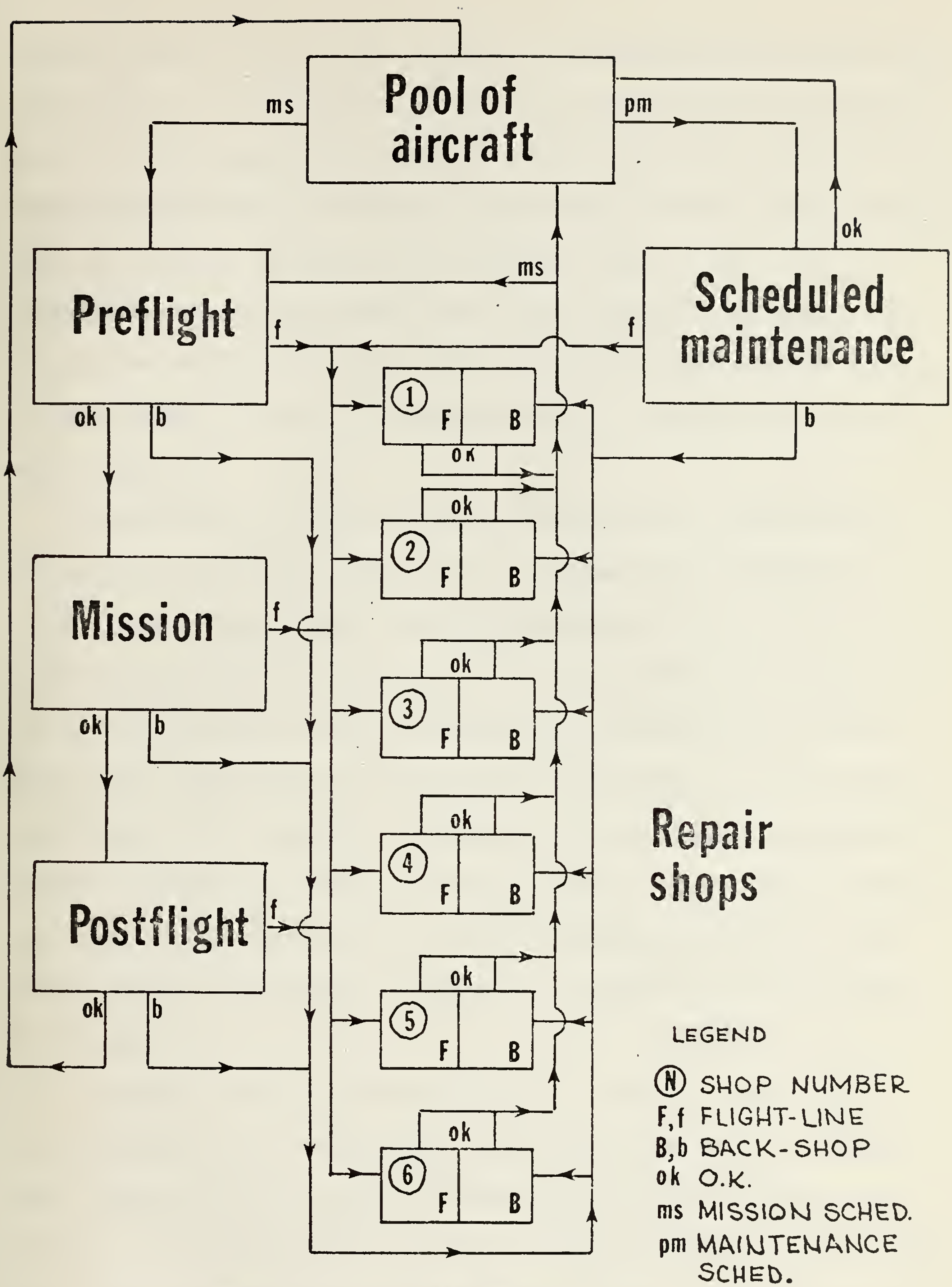
It was decided to look at a small but significant part of the total structure, and see if there were economies to be derived from the scaling-up scheme proposed in the Queen Bee and RAA structures. To restrict the analysis to the organization of repair personnel and calculate manpower savings achievable with the alternate structures appeared to be the best approach. This approach was appealing for several reasons. First, support personnel costs are a significant contributor to the total lifetime cost of a weapon system; a significant reduction here can dramatically reduce the cost of a weapon system. Second, repair organizations can be modelled as queueing systems. There are standard calculations of measures of congestion associated with queues which makes side-by-side comparison of two different repair organizations relatively simple. Third, many other costs associated with the logistics structure depend in large part on the numbers of repair personnel and on their organization and positioning within the logistics structure.

II. DESCRIPTION OF CURRENT REPAIR ORGANIZATION

Why is this thus? What is the reason of this thusness?
-Artemus Ward, Artemus Ward's Lecture

The current aircraft repair organization, hereafter referred to as an aircraft repair facility, consists of K shops, designated Shop 1, Shop 2, and so on, up to Shop K. There are two basic categories of maintenance tasks performed at each shop: flight-line repairs and back-shop repairs. The flight-line repair category consists of all those repairs that can be made on board the aircraft at the flight-line. The back-shop repair category consists of all other types of repairs. The repair facility supports an operational unit which operates a fixed number of aircraft, equal to M.

Figure 1 is a schematic diagram of the operational unit and a repair facility composed of six shops which shows how the facility and the operational unit interface. When a mission is scheduled, the required number of aircraft are removed from the pool and preflight maintenance is conducted. When preflight maintenance is satisfactorily completed, the aircraft conduct the mission, return to base where post-flight maintenance is conducted. Once this is completed, they are returned to the pool. During stand-down periods, other maintenance is conducted in addition to the preflight and postflight maintenance. This maintenance is referred to as scheduled maintenance. The portion of the diagrammed



AIRCRAFT MAINTENCE
FLOW DIAGRAM

FIGURE 1

repair shop with enclosed capital B represents the back-shop activities of the repair shop, and the portion with circled numeral and capital F indicates the flight-line activities. Small f and small b denote flight-line and back-shop failures. When a failure occurs, the cognizant shop dispatches a repairman to the aircraft where the failure occurred. If it can be repaired onboard the aircraft it is designated a flight-line failure. If not, then it is considered a back-shop failure.

Back-shop or flight-line casualties can occur during missions, preflight, postflight or scheduled preventive maintenance. We assume that casualties do not occur to aircraft that are in the pool or already down for repair. It is also assumed that the tempo of flight operations is such that the number of aircraft in the pool is very small compared to the number of aircraft available to the operational command, so that the total number of aircraft which can fail at any given time is approximately equal to the total number available, M, minus the number that are down for repair.

A repair facility consisting of K shops, each capable of performing two categories of maintenance tasks implies that there are 2K possible failures that the facility can handle. It is assumed that the 2K possible failures exhaust all of the types of failures that can occur on the aircraft. It is further assumed that all aircraft at the operational

command exhibit identical failure characteristics; all aircraft fail into the flight-line portion of Shop 1 at one constant rate, and into the flight-line of Shop 2 at a different constant rate, and so on, for all K shops. Back-shop failures exhibit the same property.

Each shop at the facility is characterized by the number of repairmen in the shop, and two repair rates, one for the flight-line repairs and the other for back-shop repairs. The repairmen are assumed to be equally capable at any task confronting the shop. This makes it possible to characterize the shop by specifying the number of repairmen, the flight-line repair rate, and the back-shop repair rate. Repairmen at one shop cannot assist repairmen at a different shop, so that if failures occur when all repairmen are busy at one shop a queue begins to form.

A priority system within each shop is in effect to insure that repairmen are assigned to flight-line repairs before commencing work on back-shop repairs. Furthermore, every repair task can be accomplished by one and only one repairman.

Let F denote the set of 2K different failure rates which characterize each of the M aircraft; then

$$F = \{\lambda_{F1}, \lambda_{B1}, \lambda_{F2}, \lambda_{B2}, \ldots, \lambda_{FK}, \lambda_{BK}\}, \qquad (2.1)$$

where the alphanumeric subscript indicates the type of failure, F and B denoting flight-line and back-shop

respectively, and the numeral denoting the cognizant repair shop number.

Let R denote the set which characterizes the repair shops. Then,

$$R = \left\{ \begin{pmatrix} S_1 \\ \mu_{F1} \\ \mu_{B1} \end{pmatrix}, \begin{pmatrix} S_2 \\ \mu_{F2} \\ \mu_{B2} \end{pmatrix}, \begin{pmatrix} S_3 \\ \mu_{F3} \\ \mu_{B3} \end{pmatrix}, \begin{pmatrix} S_4 \\ \mu_{F4} \\ \mu_{B4} \end{pmatrix}, \ldots\ldots, \begin{pmatrix} S_K \\ \mu_{FK} \\ \mu_{BK} \end{pmatrix} \right\} \qquad (2.2)$$

where S_i is the number of repairmen, μ_{Fi} is the flight-line repair rate, and μ_{Bi} is the back-shop repair rate at Shop i.

Another set of interest contains measures of congestion at the shops in the facility; it is designated P and is defined as follows:

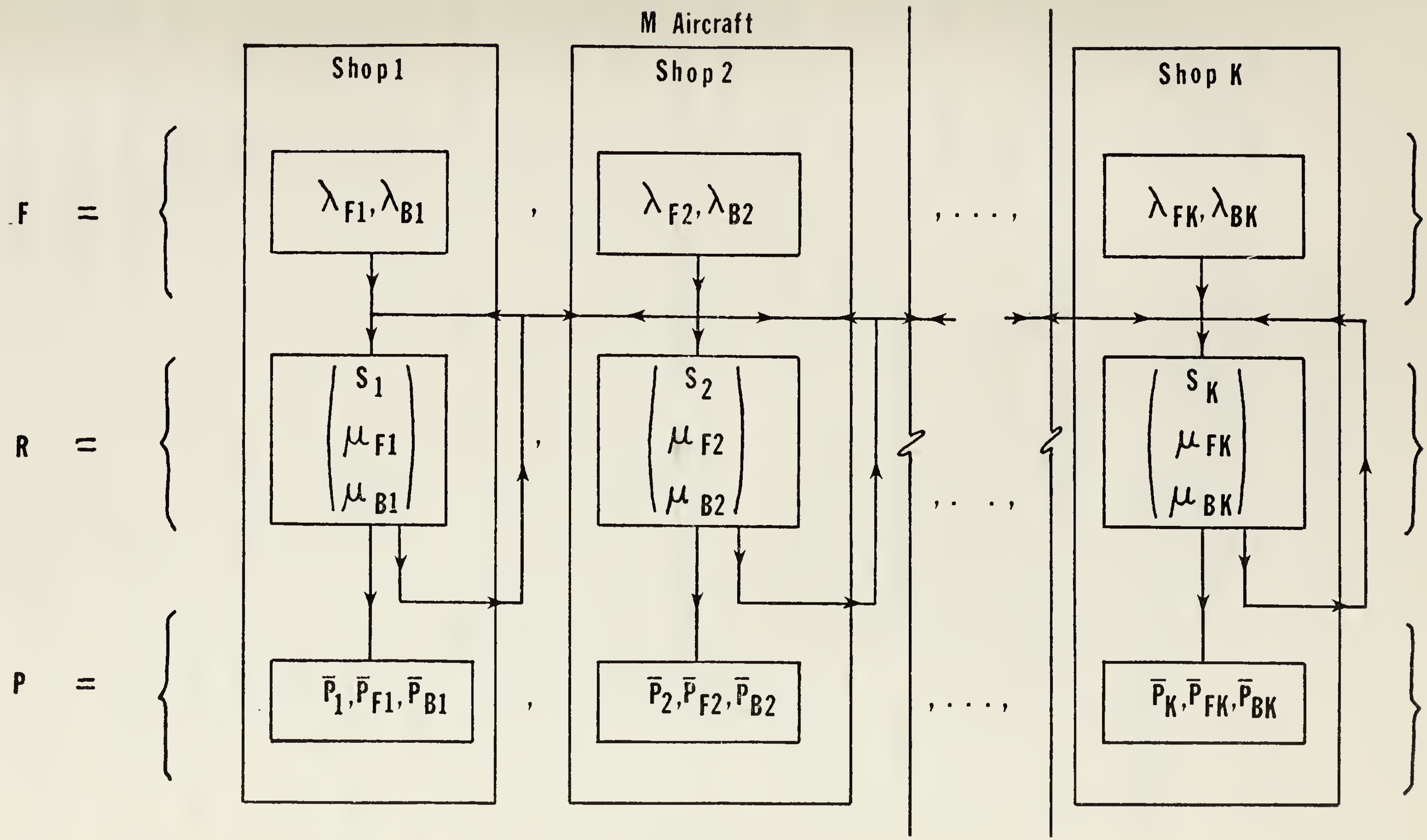
$$P = \{\overline{P}_i, \overline{P}_{Fi}, \overline{P}_{Bi}\} \quad i = 1,2,3,\ldots,K \qquad (2.3)$$

where

$$\overline{P}_i = \begin{pmatrix} E(N_i) \\ Var(N_i) \\ E(Q_i) \\ Var(Q_i) \\ E(W_i) \\ E(D_i) \end{pmatrix}, \quad \overline{P}_{Fi} = \begin{pmatrix} E(N_{Fi}) \\ Var(N_{Fi}) \\ E(Q_{Fi}) \\ Var(Q_{Fi}) \\ E(W_{Fi}) \\ E(D_{Fi}) \end{pmatrix}, \text{ and } \overline{P}_{Bi} = \begin{pmatrix} E(N_{Bi}) \\ Var(N_{Bi}) \\ E(Q_{Bi}) \\ Var(Q_{Bi}) \\ E(W_{Bi}) \\ E(D_{Bi}) \end{pmatrix}.$$

The subscript i refers to the ith shop, and the subscripts Fi and Bi refer to the flight-line and back-shop activities of the ith shop. The variable N is the number in the system, and the variable Q is the length of the queue. W is the waiting time, and D is the delay time. Waiting time is defined as the total time the aircraft is down for repair. Delay time is the time spent waiting for repairs to begin.

Figure 2 illustrates the interaction of the elements of set F with the set R, resulting in the performance measures in set P. Note the feedback loops out of R; these represent the interaction of the shops with each other. This interaction is not well understood at present, and models of the present facility developed on the following pages ignore it.



SET ELEMENT INTERACTION
IN CURRENT FACILITY
FIGURE 2

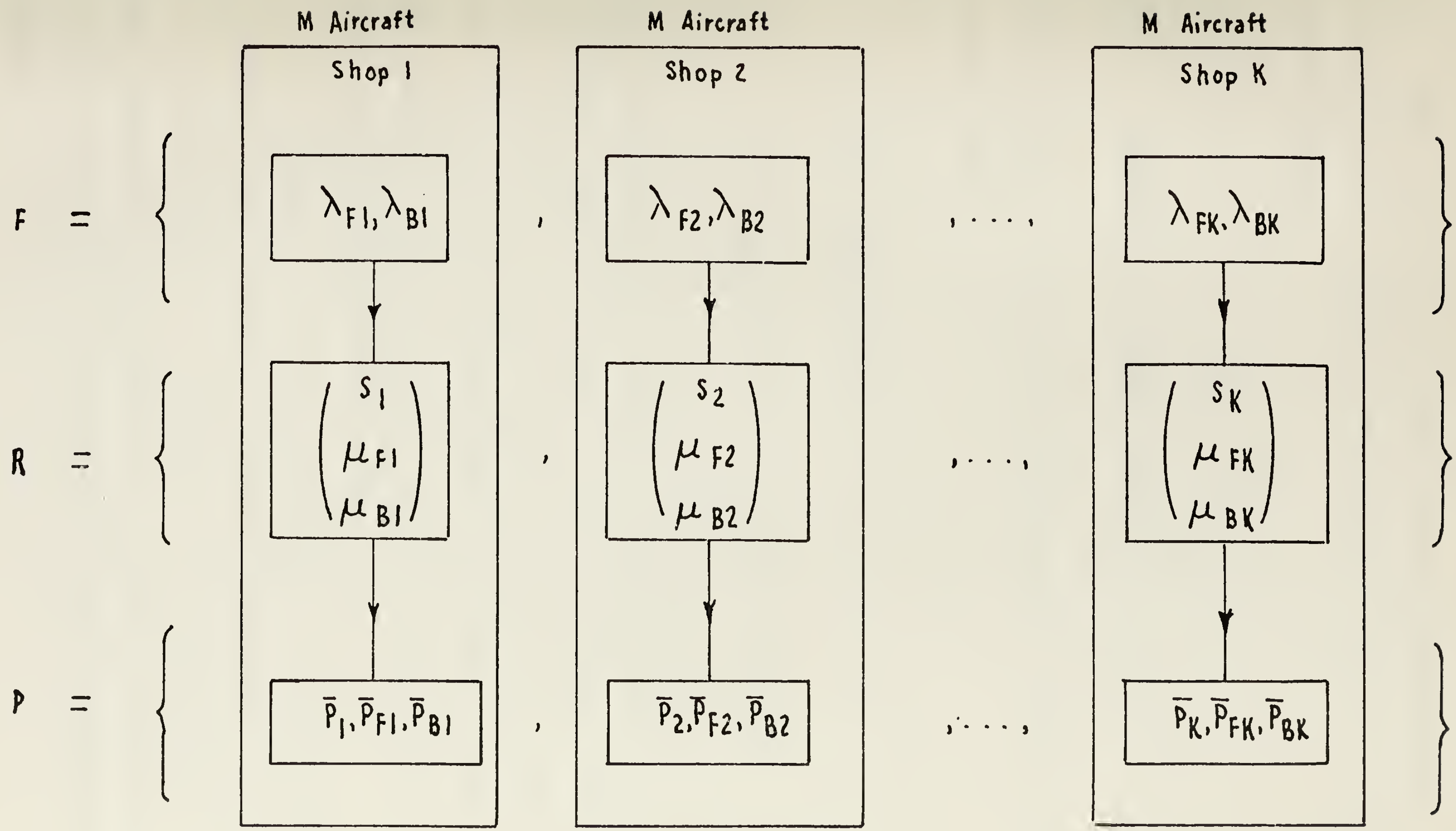
III. A BIVARIATE MODEL

How happy could I be with either,
  Were t'other dear charmer away!
But while ye thus tease me together,
  To neither a word will I say.
                    -John Gay, The Beggar's Opera

A bivariate model is developed in order to obtain values for the measures of congestion contained in the set P. The basic approach in modeling the system with a bivariate model is to treat each shop independently, and model the facility as K different queueing systems, each queueing system maintaining M aircraft. Figure 3 illustrates the approach.

Consider the queueing system defined by the interaction of Shop N with M aircraft. Assume that repair times and interarrival times are exponentially distributed. The state of the system at any time t can be represented by the ordered pair (F(t),B(t)), where F(t) is the number of aircraft down for flight-line repairs, and B(t) is the number of aircraft down for back-shop repairs at Shop N. If flight operations and maintenance are conducted throughout the 24 hour day, so that the process is not dependent on time, the bivariate model belongs to a class of probability models known as continuous-time Markov chains. These probability models are characterized by the fact that

1. They can be completely described at any time by specifying their state at that time, and



SET ELEMENT INTERACTION IN
BIVARIATE MODEL OF CURRENT FACILITY

FIGURE 3

2. the time until a transition is made from one state to a neighboring state is an exponentially distributed random variable.

Let

$$P_{fb}(t) = P[F(t) = f, \; B(t) = b]. \tag{3.1}$$

Concentrating on long-run probabilities, of more interest is the quantity

$$P_{fb} = \lim_{t\to\infty} P_{fb}(t) \tag{3.2}$$

$$= \lim_{t\to\infty} P[F(t) = f, \; B(t) = b]. \tag{3.3}$$

P_{fb} is the long-run or steady-state probability of exactly f aircraft down for repair at the flight-line, and b aircraft down for repair at the back-shop of the shop under consideration. P_{fb} is also the long-run proportion of time that there are exactly f and b aircraft down for repairs at the flight-line and back-shop respectively.

For a queueing system consisting of one shop servicing M aircraft, the possible states of the system are (f,b), $f = 1,2,3,\ldots, M$, $b = 1,2,3,\ldots, M$, such that $f + b \leq M$. The probability distribution of interest is the steady-state probabilities associated with the possible states of the system

$$\{P_{fb}\}, \quad f = 1,2,3,\ldots,M, \quad b = 1,2,3,\ldots,M,$$

$$\text{s.t.} \quad f + b \leq M.$$

The number of possible states is a quadratic function of the number of aircraft serviced by the shop. Denoting the number of possible states by N_s, the relationship is

$$N_s = \frac{(M + 1)(M + 2)}{2} . \tag{3.4}$$

The state space for a system with a large number of aircraft can be quite large, as Table I illustrates.

TABLE I

Number of Bivariate States
Vs. Number of Aircraft

M	N_s
1	3
5	21
10	66
25	351
50	1326
75	2926
100	5151
200	20301

The general principle which enables one to determine the state probabilities is derived from a rate-equality

principle which applies to continuous-time Markov chains. The principle applied to the bivariate process states that for each possible state (f,b), the rate at which the process enters the state (f,b) equals the rate at which it leaves the state (f,b). The application of this principle to each of the N_s possible states of the system will yield a set of balance equations (N_s in all) which must be solved for the set of state probabilities $\{P_{fb}\}$.

Figure 4 is a transition diagram for a hypothetical shop consisting of three repairmen with repair responsibilities for five aircraft. This shop represents the repair facility described in Chapter II. The circles represent the possible states of the system. Arrows indicate the possible transitions between states, and the labels on the arrows represent the rates at which the process moves from one state to another. The transition diagram simplifies the task of setting up the balance equations.

Balance Equations for a system of M aircraft failing into a shop with S repairmen are presented in Figure 5. These can be verified by referring to the transition diagram of Figure 4.

The solution of the system of balance equations can proceed algebraically utilizing a successive substitution scheme for small systems, but for large systems the solution becomes intractable. For this reason, an iteration algorithm was invoked so that $\{P_{fb}\}$ for systems of practical

TRANSITION DIAGRAM OF HYPOTHETICAL SHOP

Legend

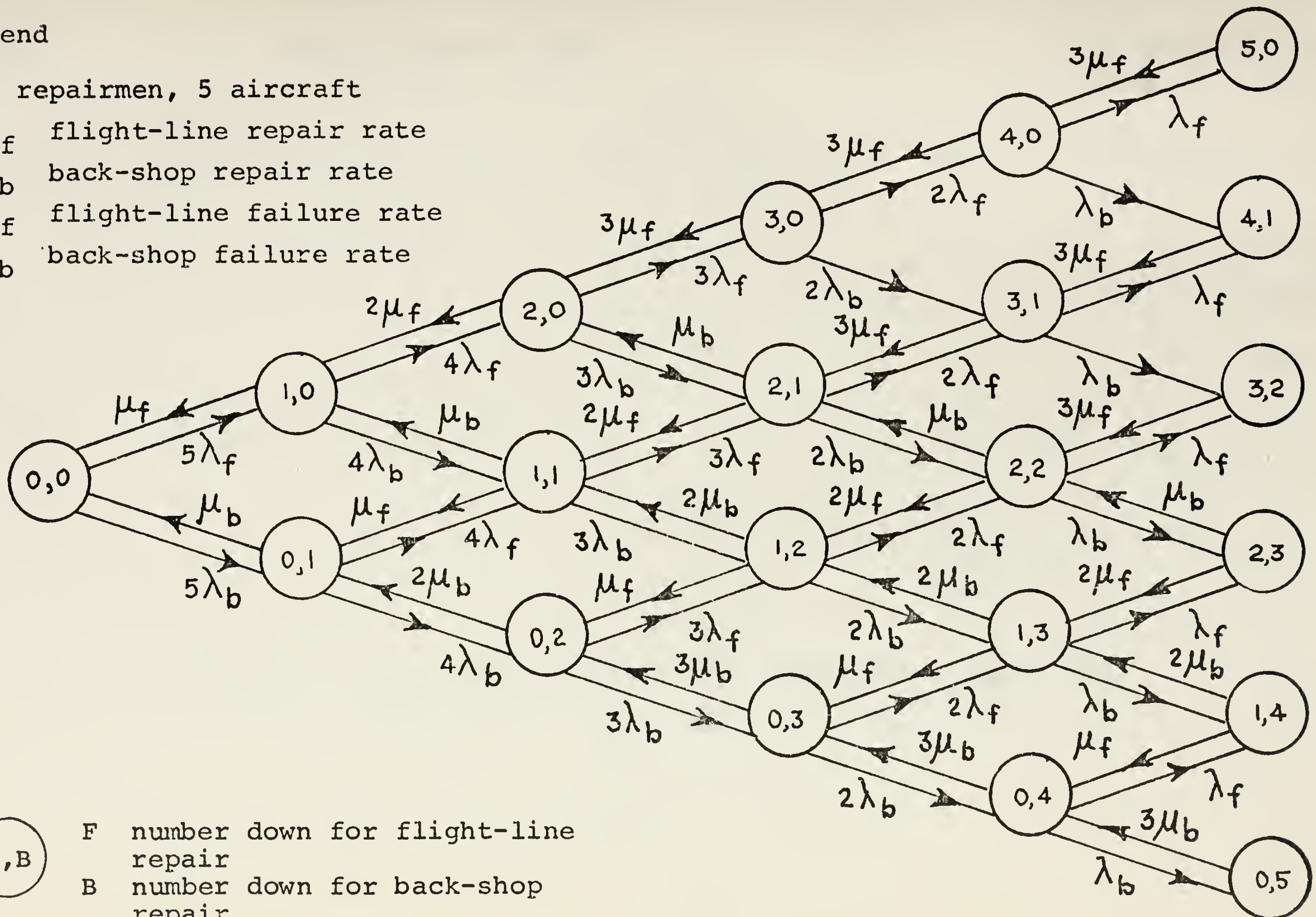
3 repairmen, 5 aircraft

μ_f flight-line repair rate

μ_b back-shop repair rate

λ_f flight-line failure rate

λ_b back-shop failure rate



F,B F number down for flight-line repair
B number down for back-shop repair

FIGURE 4

BALANCE EQUATIONS FOR THE BIVARIATE MODEL

State	Rate process leaves	=	Rate process enters
$f \leq S-1$	$\left[(M-f-b)(\lambda_b+\lambda_f) + f\mu_f + \min\begin{pmatrix} S-f \\ b \end{pmatrix}\mu_b\right] P_{fb}$	=	$(M-b-f+1)(\lambda_f P_{(f-1)b} + \lambda_b P_{f(b-1)}) + (f+1)\mu_f P_{(f+1)b} + \min\begin{pmatrix} b+1 \\ S-f \end{pmatrix}\mu_b P_{f(b+1)}$ (3.5)
$f \geq S$	$\left[(M-f-b)(\lambda_b+\lambda_f) + S\mu_f\right] P_{fb}$	=	$(M-f-b+1)(\lambda_f P_{(f-1)b} + \lambda_b P_{f(b-1)}) + S\mu_f P_{(f+1)b}$ (3.6)

$P_{fb} = 0$ for states outside the state space, i.e., for all f and b where $f+b > M$, or $f < 0$ or $b < 0$.

FIGURE 5

size could be found. The algorithm used was the Gauss-Seidel iterative method described by Varga in [Ref. 1] and by Young and Gregory in [Ref. 2]. The application of iterative algorithms to Markov-type problems is an area of active research at the Naval Postgraduate School, as it is particularly well suited for the solution of large systems of balance equations.

The algorithm as it applies to the problem at hand is set forth here without mathematical underpinnings. First, all balance equations are rearranged so that P_{fb} is expressed as a function of $P_{(f-1)b}$, $P_{(f+1)b}$, $P_{f(b-1)}$, and $P_{f(b+1)}$. This is easily done, as reference to equations (3.5) and (3.6) will show. Second, assume values for all P_{fb}. One scheme would be to assign $P_{fb} = 1/N_s$, for all f and b in the state space. Here, N_s is the number of possible states. The closer the initial assignment of values of P_{fb} is to the solution, the faster the algorithm will converge. Third, denote the assumed values of P_{fb}, $P_{fb}^{(0)}$, for the zeroth iteration. Fourth, establish a scheme for incrementing subscripts f and b. The scheme used in the problem under consideration was to start at P_{00} and increment f values by one until P_{M0} was reached, then b was incremented by one, and f values were then incremented by one, starting at f = 0 again. Thus, after P_{M0} was reached, the incrementing scheme called for P_{01}, P_{11}, P_{21}, ..., $P_{(M-1)1}$ in that order. The subscript b is again incremented by one and f values

incremented by one, so that $f = 0,1,2,\ldots, M-2$. This process is continued until all P_{fb} in the state space have been reached. This constitutes one iteration. So one iteration will consist of N_s increments. Unless we do something at each increment, of course, the algorithm would be rather pointless, so on to the remaining steps of the algorithm.

Denote the value of P_{fb} computed at the nth iteration $P_{fb}^{(n)}$. Recall that all balance equations were rearranged so that P_{fb} equals some function of $P_{(f-1)b}$, $P_{(f+1)b}$, $P_{f(b-1)}$, $P_{f(b+1)}$. Each balance equation will generate a different function, but to keep the notation simple, denote that function generated by the balance equation under consideration g, where it is understood that the gs will be different for each equation. We have

$$P_{fb} = g(P_{(f-1)b}, P_{(f+1)b}, P_{f(b-1)}, P_{f(b+1)}). \tag{3.7}$$

Define

$$P_{fb}^{(n)} = g(P_{(f-1)b}^{(n-1)}, P_{(f+1)b}^{(n-1)}, P_{f(b-1)}^{(n-1)}, P_{f(b+1)}^{(n-1)}) \tag{3.8}$$

as the nth iteration approximation of P_{fb}. Note that it is computed from the (n-1)th iteration approximations of $P_{(f-1)b}$, $P_{(f+1)b}$, $P_{f(b-1)}$, and $P_{f(b+1)}$.

The nth iteration values of P_{fb} are computed as follows:

1. For the first P_{fb} in the incrementing scheme, compute $P_{fb}^{(n)}$ in accordance with equation (3.8).

2. Immediately change $P_{fb}^{(n-1)}$ to $P_{fb}^{(n)}$. For example, if $P_{00}^{(1)}$ is being computed, then immediately after the step where a value for $P_{00}^{(1)}$ is obtained, change $P_{00}^{(0)}$ to that value. Now, when the incrementing scheme calls for the computation of a $P_{fb}^{(1)}$ involving a $P_{00}^{(0)}$, the algorithm will use the most recent approximation of P_{00}, which is the value computed for $P_{00}^{(1)}$.

3. Continue through the incrementing scheme, doing steps 1 and 2, until the algorithm has incremented through all possible states.

4. Make the following computation:

$$S = \sum_{b=0}^{M} \sum_{f=0}^{M-b} P_{fb}^{(n)} \tag{3.9}$$

5. Normalize all of the $P_{fb}^{(n)}$ by dividing each one by the value of S obtained in step 4 above. The $P_{fb}^{(n)}$'s now sum to one.

6. The values of the $P_{fb}^{(n)}$ just obtained are used in the computation of $P_{fb}^{(n+1)}$ during the (n+1)th iteration, which proceeds exactly the same as the nth, following steps 1 through 5.

Eventually, the algorithm will converge on the solution of the system of balance equations.[1] The bivariate model balance equations were solved in the above manner. The average number of iterations required for convergence was approximately forty-to-fifty, with one system converging in seventeen iterations. The end result of the algorithm is a solution for $\{P_{fb}\}$ which was used to calculate the measures of congestion contained in the set P (equation (2.3)).

The calculation of the elements of P is straight forward. Each component of the vector elements $\overline{P}_i$, $\overline{P}_{Fi}$, and $\overline{P}_{Bi}$ is a standard queueing system calculation. The computation of each component proceeds without difficulty, once the set $\{P_{fb}\}$ is known.

Define $\{P_f\}$ as the univariate probability distribution of the flight-line portion of the queueing system under consideration. Then

$$\{P_f\} = \{P[F = f]\}, \quad f = 0,1,2,\ldots,M , \qquad (3.10)$$

where

---

[1] The fact that the Gauss-Seidel iterative method described converges has been established by J.P. Lehoczky (private communication).

$$P_f = P[F = f] = \sum_{b=0}^{M-f} P_{fb} \qquad (3.11)$$

Similarly, for the back-shop,

$$P_b = P[B = b], \quad b = 0,1,2,\ldots,M \,, \qquad (3.12)$$

where

$$P_b = P[B = b] = \sum_{f=0}^{M-b} P_{fb} \,. \qquad (3.13)$$

The univariate probability distribution of the number of aircraft in the queueing system in the aggregate (flight-line and back-shop) is defined as P_n, where

$$P_n = P[N = n], \quad n = 0,1,2,\ldots,M \,, \qquad (3.14)$$

with

$$P_n = P[N = n] = \sum_{f=0}^{n} P_{f,(n-f)} \qquad (3.15)$$

or

$$P_n = \sum_{b=0}^{n} P_{(n-b),b} \,. \qquad (3.16)$$

The expected number of aircraft down for flight-line repair at the queueing system under consideration (shop i) is

$$E[N_{Fi}] = \sum_{f=1}^{M} f\, P_f \tag{3.17}$$

where P_f is the value obtained from equation (3.11). Similarly for the back-shop,

$$E[N_{Bi}] = \sum_{b=1}^{M} b\, P_b \tag{3.18}$$

where P_b is the value obtained from equation (3.13). The expected number of aircraft down in the system as a whole is

$$E[N_i] = \sum_{n=1}^{M} n\, P_n \tag{3.19}$$

where P_n is obtained from either equation (3.15) or equation (3.16).

The expected number of aircraft in the flight-line queue is

$$E[Q_{Fi}] = \sum_{f=S_i+1}^{M} (f - S_i)\, P_f \quad . \tag{3.20}$$

The expected number of aircraft in the back-shop queue is obtained after some head scratching. When the number of

aircraft that are down on the flight-line is less than or equal to the number of repairmen at the shop ($f \leq S_i$), f repairmen will be working at the flight-line, with the remaining $S_i - f$ men available for back-shop repairs. As long as the number of aircraft down for back-shop repairs, b, is less than or equal to $s_i - f$, no back-shop queue will exist, since there is an adequate number of repairmen available to handle back-shop tasks. This implies that the queue at the back-shop exists when $b \geq S_i - f + 1$. The queue will contain $(f + b - S_i)$ aircraft. Thus the expected number in queue at the back-shop when $0 \leq f \leq S_i$ is

$$\sum_{f=0}^{S_i} \sum_{b=S_i-f+1}^{M-f} (b+f-S_i) P_{fb} .$$

When $f \geq S_i$, all repair personnel are tied-up with flight-line repairs, so that every aircraft down for back-shop repair is in queue, awaiting service. This yields

$$\sum_{f=S_i+1}^{M-1} \sum_{b=1}^{M-f} b P_{fb} .$$

Adding the two expressions together gives the expected number in the back-shop queue:

$$E[Q_{Bi}] = \sum_{f=0}^{S_i} \sum_{b=S_i+1-f}^{M-f} (b + f - S_i) P_{fb}$$

$$+ \sum_{f=S_i+1}^{M-1} \sum_{b=1}^{M-f} b P_{fb} . \tag{3.21}$$

The expected number of aircraft in queue for the system as a whole is similar to the back-shop queue in regard to derivation. When $f \le S_i$, the only aircraft in queue are those in the back-shop queue, since there are enough repairmen to handle flight-line repairs. Now, when $f > S_i$, queues can exist at both the flight-line and the back-shop. The number of aircraft in queue will be $(f + b - S_i)$. This yields

$$E[Q_i] = \sum_{f=0}^{S_i} \sum_{b=S_i+1-f}^{M-f} (b + f - S_i)\, P_{fb}$$

$$+ \sum_{f=S_i+1}^{M} \sum_{b=0}^{M-f} (b + f - S_i)\, P_{fb}. \qquad (3.22)$$

The variances of the number of aircraft in the flight-line, back-shop, and total system are $Var[N_{Fi}]$, $Var[N_{Bi}]$, and $Var[N_i]$, where

$$Var[N_{Fi}] = E[N_{Fi}^2] - (E[N_{Fi}])^2 \qquad (3.23)$$

$$= \sum_{f=1}^{M} f^2\, P_f - [\sum_{f=1}^{M} f\, P_f]^2 , \qquad (3.24)$$

$$Var[N_{Bi}] = E[N_{Bi}^2] - (E[N_{Bi}])^2 \qquad (3.25)$$

$$= \sum_{b=1}^{M} b^2\, P_b - [\sum_{b=1}^{M} b\, P_b]^2 , \qquad (3.26)$$

and

$$\mathrm{Var}[N_i] = E[N_i^2] - (E[N_i])^2 \tag{3.27}$$

$$\sum_{n=1}^{M} n^2 P_n - [\sum_{n=1}^{M} n P_n]^2 . \tag{3.28}$$

The variance of the number of aircraft in queue at the flight-line is

$$\mathrm{Var}[Q_{Fi}] = E[Q_{Fi}^2] - ([Q_{Fi}])^2 \tag{3.29}$$

$$= \sum_{f=S_i+1}^{M} (f-S_i)^2 P_f - [\sum_{f=S_i+1}^{M} (f-S_i) P_f]^2 . \tag{3.30}$$

The variance of the number of aircraft in queue at the back-shop is

$$\mathrm{Var}[Q_{Bi}] = E[Q_{Bi}^2] - (E[Q_{Bi}])^2 \tag{3.31}$$

$$= \sum_{f=0}^{S_i} \sum_{b=S_i+1-f}^{M-f} (b+f-S_i)^2 P_{fb} + \sum_{f=S_i+1}^{M-1} \sum_{b=1}^{M-f} b^2 P_{fb} - (E[Q_{Bi}])^2 . \tag{3.32}$$

The variance of the number of aircraft in queue at the system as a whole is

$$\mathrm{Var}[Q_i] = E[Q_i^2] - (E[Q_i])^2 \tag{3.33}$$

$$= \sum_{f=0}^{S_i} \sum_{b=S_i+1-f}^{M-f} (b+f-S_i)^2 \; P_{fb}$$

$$+ \sum_{f=S_i+1}^{M} \sum_{b=0}^{M-f} (f+b-S_i)^2 \; P_{fb}$$

$$- (E[Q_i])^2 \; . \tag{3.34}$$

The calculation of the expected waiting times $E[W_{Fi}]$, $E[W_{Bi}]$ and $E[W_i]$ is based on the queueing relationship "$L = \lambda W$", where L is the expected number of customers in the system, λ is the average arrival rate, and W is the expected waiting time. Denoting the average arrival rates at the flight-line and back-shop by λ_{Fa} and λ_{Ba}, respectively, and the average arrival rate to the total system by λ_a, the queueing relationship is restated in terms of notation applicable to the problem at hand. Thus, we have

$$E[N_{Fi}] = \lambda_{Fa} \, E[W_{Fi}], \tag{3.35}$$

$$E[N_{Bi}] = \lambda_{Ba} \, E[WB_i], \tag{3.36}$$

and

$$E[N_i] = \lambda_a E[W_i]. \tag{3.37}$$

Given that the state of the system is (f,b) there are exactly (M - f - b) aircraft that can fail into the system. They each have individual failure rates λ_{Fi}, so that the failure rate into the system in state (f,b) is $(M - f - b)\lambda_{Fi}$. The proportion of time that the system is in state (f,b) is P_{fb}. Summing up the failure rate into the system at state (f,b) times the proportion of the time that the system is in state (f,b) over all possible states yields the average failure rate for the system under consideration. Thus,

$$\lambda_{Fa} = \sum_{b=0}^{M-f} \sum_{f=0}^{M} (M-f-b)\ \lambda_{Fi}\ P_{fb} \tag{3.37}$$

$$= \lambda_{Fi} \sum_{b=0}^{M-f} \sum_{f=0}^{M} M\ P_{fb} - \lambda_{Fi} \sum_{b=0}^{M-f} \sum_{f=0}^{M} (f + b)\ P_{fb}$$

$$= \lambda_{Fi} [M \sum_{b=0}^{M-f} \sum_{f=0}^{M} P_{fb} - \sum_{b=0}^{M-f} \sum_{f=0}^{M} f\ P_{fb} - \sum_{b=0}^{M-f} \sum_{f=0}^{M} b\ P_{fb}]$$

$$= \lambda_{Fi}\ [M - E[N_{Fi}] - E[N_{Bi}]]. \tag{3.38}$$

$$\lambda_{Fa} = \lambda_{Fi}\ [M - E[N_i]]\ . \tag{3.39}$$

Similarly,

$$\lambda_{Ba} = \lambda_{Bi} [M - E[N_i]] . \tag{3.40}$$

Derivation of the overall system average arrival rate yields

$$\lambda_a = \sum_{b=0}^{M-f} \sum_{f=0}^{M} (M - f - b)(\lambda_{Fi} + \lambda_{Bi}) P_{fb} \tag{3.41}$$

$$= \sum_{b=0}^{M-f} \sum_{f=0}^{M} (M - f - b) \lambda_{Fi} P_{fb}$$

$$+ \sum_{b=0}^{M-f} \sum_{f=0}^{M} (M - f - b) \lambda_{Bi} P_{fb}$$

$$= \lambda_{Fa} + \lambda_{Ba} \tag{3.42}$$

which is no surprise.

The calculation of delay times is based on the queueing relationship $Q = \lambda D$. Stated in terms of applicable notation the delay times are:

$$E[Q_{Fi}] = \lambda_{Fa} E[D_{Fi}] , \tag{3.43}$$

$$E[Q_{Bi}] = \lambda_{Ba} E[D_{Bi}] , \tag{3.44}$$

and

$$E[Q_i] = \lambda_a E[D_i] . \tag{3.45}$$

The bivariate model is an approximate model of the current aircraft repair facility described in Chapter II. The purpose of the development of the model is to provide a basis of comparison for scaled-up versions such as the RAA and the Queen Bee structures. Two other approximate models, called decomposition models, were developed to assist in modelling the scaled-up structures. A discussion of these models is presented in the next chapter.

IV. TWO DECOMPOSITION MODELS

He was in logic a great critic,
Profoundly skilled in analytic.
He could distinguish, and divide ...
-Samuel Butler, Hudibras

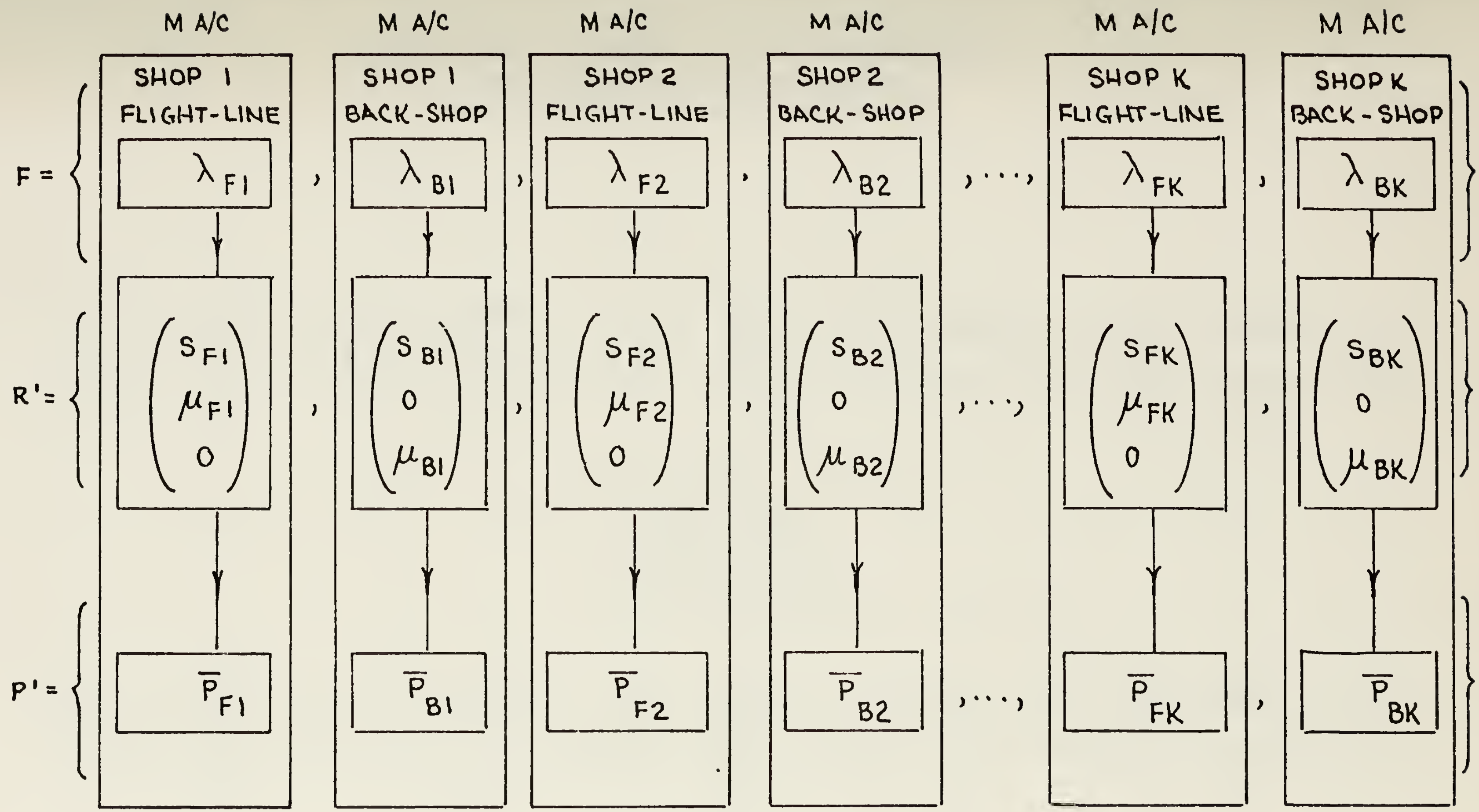
Initially conceived to provide a rough-and-ready model of the current repair facilities, the decomposition models were soon abandoned in favor of the more refined bivariate model described in the last chapter. However, if scaling-up is to occur, the current repair shops must be physically decomposed in some manner, as repair personnel will be taken from shops as currently organized and reassigned into Queen Bee or RAA Support Mission Bases. One scheme is to maintain sufficient personnel at each shop in the facility to handle flight-line repairs only, and reassign the remainder to an aggregated facility with exclusive responsibility for back-shop repairs. Men would be drawn from several bases for the aggregated back-shop. The idea is that the pooled back-shop resources from the bases would provide more than sufficient manpower to give adequate back-shop support. The excess manpower would then represent the manpower savings achieved by reorganization into the new structure.

Part of the problem is to determine how many men to leave at the flight-line facilities. The bivariate model helps in this, since it provides a measure of flight-line performance at the shops represented by the vectors $\overline{P}_{Fi}$,

$i = 1,2,3,\ldots,K$. In addition, the bivariate model gives measures of back-shop performance in the vectors $\bar{P}_{Bi}$, $i = 1,2,3,\ldots,K$. A decomposition model fits in nicely at this juncture in the overall problem. With a model that yields measures of performance for various allocations of shop personnel between exclusive flight-line duties and exclusive back-shop duties, all that is needed is to find that allocation that meets present flight-line performance criteria, and the numbers available for the pooled back-shop are in hand. The back-shop can then be modeled, utilizing one of the simpler queueing models, to find the number of personnel required to provide comparable support to that provided by the current facility, and the manpower not needed represents the savings acquired by reorganization.

Figure 6 best describes the decomposition approach. Again, independence between shops and exponentially distributed repair and interarrival times are assumed. A further assumption is that flight-line and back-shop activities within each shop are independent of each other. The S_i men assigned to shop i are allocated so that S_{Fi} men conduct flight-line repairs exclusively, with the remaining S_{Bi} working in the back-shop.

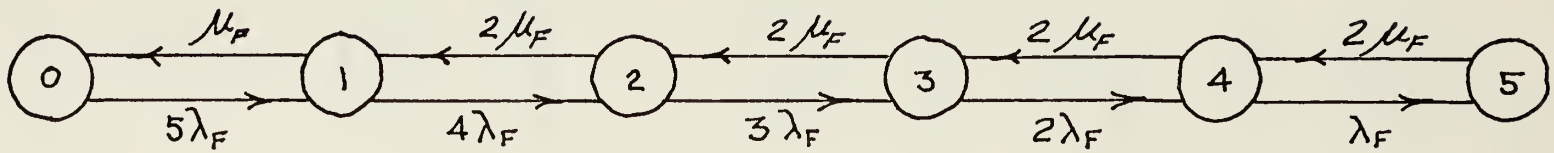
Figure 7 is a transition diagram of the process. At each shop, two independent processes operate simultaneously. Each process is identical to that described by Gaver and Thompson in [Ref. 3] as the Finite Arrival Source, or



SET ELEMENT INTERACTION IN
DECOMPOSITION MODEL OF CURRENT FACILITY

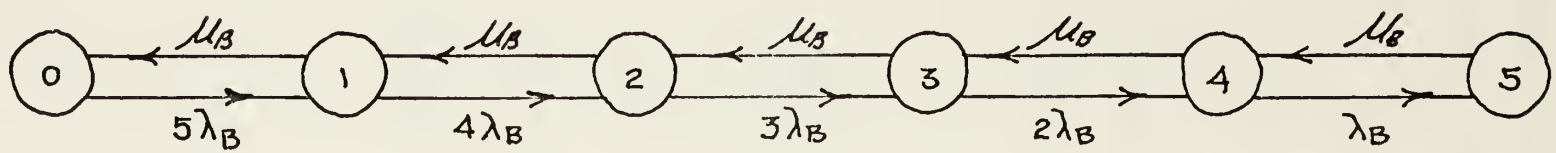
FIGURE 6

TRANSITION DIAGRAM OF HYPOTHETICAL
REPAIR SHOP MODELED AS TWO INDEPENDENT
FINITE ARRIVAL SOURCE QUEUES
DECOMPOSITION REPAIRMAN MODEL



Flight-line

5 aircraft
2 repairmen
λ_f arrival rate
μ_f service rate



Back-shop

5 aircraft
1 repairman
λ_b arrival rate
μ_b service rate

FIGURE 7

Repairman problem. The state of each process can be represented by $(F(t))$, or $(B(t))$ as applicable. $F(t)$ is the number down for repairs at the flight-line, and $B(t)$ is the number down for back-shop repairs. Let

$$P_f(t) = P[F(t) = f] \ . \tag{4.1}$$

Again, the long-run probabilities are the sought quantities, i.e.

$$\begin{aligned} P_f &= \lim_{t\to\infty} [P_f(t)] \\ &= \lim_{t\to\infty} P[F(t) = f] \ , \end{aligned} \tag{4.2}$$

and

$$P_b = \lim_{t\to\infty} P[B(t) = b] \ . \tag{4.3}$$

Balance equations for the queueing system represented by flight-line of Shop i are:

State	Rate Process Leaves	=	Rate Process Enters	
0	$M\lambda_{Fi}P_0$	=	$\mu_{Fi}P_1$	(4.4)
1	$[(M-1)\lambda_{Fi} + \mu_{Fi}]P_1$	=	$M\lambda_{Fi}P_0 + 2\mu_{Fi}P_2$	(4.5)
2	$[(M-2)\lambda_{Fi} + 2\mu_{Fi}]P_2$	=	$(M-1)\lambda_{Fi}P_1\mu + 3\ {}_{Fi}P_3$	(4.6)
$\vdots$	$\vdots$		$\vdots$	
$0 < f < S_{Fi}$	$[(M-f)\lambda_{Fi} + f\mu_{Fi}]P_f$	=	$(M-f+1)\lambda_{Fi}P_{f-1} + (f+1)\mu_{Fi}P_{f+1}$	(4.7)
$S_{Fi} \leq f < M$	$[(M-f)\lambda_{Fi} + S_{Fi}\mu_{Fi}]P_f$	=	$(M-f+1)\lambda_{Fi}P_{f-1} + S_{Fi}\mu_{Fi}P_{f+1}$	(4.8)
M	$S_{Fi}\mu_{Fi}P_M$	=	$\lambda_{Fi}P_{M-1}$.	(4.9)

Replacing the term $M\lambda_{Fi}P_0$ on the left side of the balance equation for state 1 with its equivalent $\mu_{Fi}P_1$ obtained from the balance equation for state 0, the balance equation for state 1 becomes

$$(M-1)\lambda_{Fi} + \mu_{Fi}\ P_1 = \mu_{Fi}\ P_1 + 2\mu_{Fi}\ P_2\ . \tag{4-10}$$

Clearing terms ,

$$(M-1)\lambda_{Fi}\ P_1 \ = \ 2\mu_{Fi}\ P_2 \quad . \tag{4.11}$$

Replacing the expression $(M-1)\lambda_{Fi}P_1$ on the left side of the balance equation for state 2 with its equivalent just found in equation (4.11) $2\mu_{Fi}P_2$, and clearing terms, the balance equation for state 2 becomes

$$(M-2)\lambda_{Fi}\ P_2 \ = \ 3\mu_{Fi}\ P_3 \quad . \qquad (4-12)$$

This process continued down the states from state 3 to state M yields the following set of balance equations:

$$\lambda_f P_f \ = \ \mu_{f+1}P_{f+1} \ , \qquad 0 \leq f \leq M-1 \tag{4.13}$$

where

$$\lambda_f \ = \ (M-f)\lambda_{Fi} \ , \qquad 0 \leq f \leq M-1 \tag{4.14}$$

$$\mu_f \ = \ f\mu_{Fi} \ , \qquad 0 < f \leq S_{Fi} \qquad (4.15$$

$$= \ S_{Fi}\mu_{Fi} \ , \qquad S_{Fi} \leq f \leq M \quad . \tag{4.16}$$

The solution to this set of balance equations is

$$P_f \ = \ P_0 \prod_{j=1}^{f} \frac{\lambda_{j-1}}{\mu_j} \ , \qquad 0 < f \leq M \quad . \tag{4.17}$$

P_0 is obtained from the requirement that

$$\sum_{f=0}^{M} P_f = 1 \; . \tag{4.18}$$

Thus,

$$P_0 + \sum_{f=1}^{M} P_f = \sum_{f=0}^{M} P_f = 1 \; ,$$

$$P_0 + \sum_{f=1}^{M}\left[P_0 \prod_{j=1}^{f} \frac{\lambda_{j-1}}{\mu_j} \right] = 1 \; ,$$

and

$$P_0 \left[1 + \sum_{f=1}^{M}\left[\prod_{j=1}^{f} \frac{\lambda_{j-1}}{\mu_j} \right] \right] = 1 \; . \tag{4.19}$$

The solution of the balance equations obtained by substituting expressions for λ_f and μ_f obtained from equations (4.14), (4.15) and (4.16) into equations (4.17) and (4.19) yields the following solution for the flight-line balance equations:

$$P_0 = \frac{1}{1 + \sum_{f=1}^{S_{Fi}} \frac{M!}{(M-f)!f!} \left(\frac{\lambda_{Fi}}{\mu_{Fi}} \right)^f + \sum_{f=S_{Fi}+1}^{M} \frac{M!}{(M-f)!S_{Fi}!S_{Fi}^{\,f-S_{Fi}}} \left(\frac{\lambda_{Fi}}{\mu_{Fi}} \right)^f} \tag{4.20}$$

$$P_f = \frac{M!}{(M-f)!f!} \left(\frac{\lambda_{Fi}}{\mu_{Fi}} \right)^f P_0 \qquad , \quad 0 < f \le S_{Fi} \tag{4.21}$$

$$P_f = \frac{M!}{(M-f)!\, S_{Fi}!\, S_{Fi}^{\,f-S_{Fi}}} \left(\frac{\lambda_{Fi}}{\mu_{Fi}}\right)^f P_0 \quad , \quad S_{Fi} \le f \le M \,. \tag{4.22}$$

The solution for the back-shop balance equations is obtained employing the same method used to solve the flight-line balance equations. The solution is

$$P_0 = \frac{1}{1 + \sum_{b=1}^{S_{Bi}} \frac{M!}{(M-b)!\,b!} \left(\frac{\lambda_{Bi}}{\mu_{Bi}}\right)^b + \sum_{b=S_{Bi}+1}^{M} \frac{M!}{(M-b)!\,S_{Bi}!\,S_{Bi}^{\,b-S_{Bi}}} \left(\frac{\lambda_{Bi}}{\mu_{Bi}}\right)^b} \,, \tag{4.23}$$

$$P_b = \frac{M!}{(M-b)!\,b!} \left(\frac{\lambda_{Bi}}{\mu_{Bi}}\right)^b P_0 \quad , \quad 0 < b \le S_{Bi} \tag{4.24}$$

and

$$P_b = \frac{M!}{(M-b)!\,S_{Bi}!\,S_{Bi}^{\,b-S_{Bi}}} \left(\frac{\lambda_{Bi}}{\mu_{Bi}}\right)^b P_0 \quad , \quad S_{Bi} \le b \le M \tag{4.25}$$

Calculation of the elements of $\overline{P}_{Fi}$ and $\overline{P}_{Bi}$ were made utilizing equations from Chapter III where appropriate. For the expected number of aircraft down, equations (3.17), (3.18), and (3.19) were used. The expected number of aircraft in queue at the flight-line was calculated from equation (3.20).

The expected number of aircraft in the back-shop queue was calculated using

$$E(Q_{Bi}) = \sum_{b=S_i+1}^{M} (b - S_i)\ P_b \qquad (4.26)$$

The variance of the number of aircraft in the flight-line and back-shop were calculated using equations (3.24) and (3.26), respectively. The variance of the number of aircraft in flight-line queue was computed from equation (3.30). The variance of the back-shop queue was computed from

$$\mathrm{Var}(Q_{Bi}) = \sum_{b=S_i+1}^{M} (b - S_I)^2\ P_b - (E[Q_{Bi}])^2 \quad . \qquad (4.27)$$

Calculation of the expected waiting times and expected delay times were made utilizing equations (3.35), (3.36), (3.43), and (3.44) with

$$\lambda_{Fa} = \lambda_{Fi} \left[M - E[N_{Fi}] \right] \qquad (4.28)$$

and

$$\lambda_{Ba} = \lambda_{Bi} \left[M - E[N_{Bi}] \right] \quad . \qquad (4.29)$$

Another decomposition model assumes a Poisson arrival process, with constant arrival rate $M\lambda_{Fi}$ or $M\lambda_{Bi}$ as applicable. Although not as accurate as the repairman model, it can be applied to systems with a large number of aircraft and

sufficiently manned so that the number of aircraft operational is kept at high levels.

Again the problem has been reduced to the repeated solution of a simple model, this time the Poisson-Arrival, Parallel-server, Exponential Service Time System, or M/M/S Queue described by Gaver and Thompson in Ref. 4. The long-run probability distributions $\{P_f\}$ and $\{P_b\}$ can be found, utilizing

$$P_f = \frac{P_{f0}}{f!} \left(\frac{M\lambda_{Fi}}{\mu_{Fi}} \right)^f \quad , \quad 0 \leq f \leq S_{Fi} \tag{4.30}$$

and

$$= \frac{P_{f0}}{S_{Fi}!\, S_{Fi}^{\,f-S_{Fi}}} \left(\frac{M\lambda_{Fi}}{\mu_{Fi}} \right)^f \quad , \quad S_{Fi} \leq f \tag{4.31}$$

where

$$P_{f0} = \left\{ \sum_{f=0}^{S_{Fi}} \frac{1}{f!} \left(\frac{M\lambda_{Fi}}{\mu_{Fi}} \right)^f + \left(\frac{M\lambda_{Fi}}{\mu_{Fi}} \right)^{S_{Fi}} \left(\frac{1}{S_{Fi}!} \right) \left(\frac{\dfrac{M\lambda_{Fi}}{S_{Fi}\mu_{Fi}}}{1 - \dfrac{M\lambda_{Fi}}{S_{Fi}\mu_{Fi}}} \right) \right\}^{-1} . \tag{4 32}$$

For the back-shop, use

$$P_b = \frac{P_{b0}}{b!}\left(\frac{M\lambda_{Bi}}{\mu_{Bi}}\right)^b \quad , \quad 0 \le b \le S_{Bi} \tag{4.33}$$

$$= \frac{P_{b0}}{S_{Bi}!\ S_{Bi}^{\ b-S_{Bi}}}\left(\frac{M\lambda_{Bi}}{\mu_{Bi}}\right)^b \quad , \quad S_{Bi} \le b \tag{4.34}$$

where

$$P_{b0} = \left\{ \sum_{b=0}^{S_{Bi}} \frac{1}{b!}\left(\frac{M\lambda_{Bi}}{\mu_{Bi}}\right)^b + \left(\frac{M\lambda_{Bi}}{\mu_{Bi}}\right)^{S_{Bi}}\left(\frac{1}{S_{Bi}!}\right)\left(\frac{\frac{M\lambda_{Bi}}{S_{Bi\ Bi}}}{1 - \frac{M\lambda_{Bi}}{S_{Bi}\mu_{Bi}}}\right)\right\}^{-1} \tag{4.35}$$

The expected number of aircraft down for repairs at the flight-line and back shop are

$$E[N_{Fi}] = \frac{M\lambda_{Fi}}{\mu_{Fi}} + P_{f0}\left(\frac{M\lambda_{Fi}}{\mu_{Fi}}\right)^{S_{Fi}}\left(\frac{1}{S_{Fi}!}\right)\left\{\frac{\frac{M\lambda_{Fi}}{S_{Fi}\mu_{Fi}}}{\left(1 - \frac{M\lambda_{Fi}}{S_{Fi}\mu_{Fi}}\right)^2}\right\}, \tag{4.36}$$

and

$$E[N_{Bi}] = \frac{M\lambda_{Bi}}{\mu_{Bi}} + P_{b0}\left(\frac{M\lambda_{Bi}}{\mu_{Fi}}\right)^{S_{Bi}}\left(\frac{1}{S_{Bi}!}\right)\left\{\frac{\frac{M\lambda_{Bi}}{S_{Bi}\mu_{Bi}}}{\left(1 - \frac{M\lambda_{Bi}}{S_{Bi}\mu_{Bi}}\right)^2}\right\}. \tag{4.37}$$

The expected queue lengths are

$$E[Q_{Fi}] = E[N_{Fi}] - \frac{M\lambda_{Fi}}{\mu_{Fi}} \tag{4.38}$$

and

$$E[Q_{Bi}] = E[N_{Bi}] - \frac{M\lambda_{Bi}}{\mu_{Bi}} \tag{4.39}$$

The expected delay times are

$$E[D_{Fi}] = \frac{E[Q_{Fi}]}{M\lambda_{Fi}} \tag{4.40}$$

and

$$E[D_{Bi}] = \frac{E[Q_{Bi}]}{M\lambda_{Bi}} . \tag{4.41}$$

The expected waiting times are

$$E[W_{Fi}] = E[D_{Fi}] + \frac{1}{\mu_{Fi}} \tag{4.42}$$

and

$$E[W_{Bi}] = E[D_{Bi}] + \frac{1}{\mu_{Bi}} . \tag{4.43}$$

V. COMPARISON OF MODELS

Comparisons are odorous.
-William Shakespeare, Much Ado About Nothing

FORTRAN programs were written to obtain solutions for the bivariate and decomposition (repairman) models. Results for the decomposition (M/M/S) model were obtained from previous M/M/S queue calculations made at RAND. Data used in the programs was provided by RAND. It consisted of an arrival rate, a service rate, and the number of servers at each shop of a 20 shop repair facility. It was decided to work with only 10 shops of the 20. This decision eliminated unnecessary repetitive calculations which would have added little to the model comparisons, and still provided enough data so that the manpower savings resulting from a consolidation of back-shop functions could be examined.

The data could not be directly applied to the models, since arrival rates and service rates were computed from all repair transactions at each shop, without differentiation as to category of repair, flight-line or back-shop. A rough estimate of the ratio of back-shop to flight-line repairs (.25) provided a basis for converting the arrival rate into the shop into flight-line and back-shop arrival rates. Back-shop and flight-line service rates were estimated to be equal, so no conversion was necessary. The number of aircraft used in the decomposition models was 25. Table II shows how the data was converted.

TABLE II

Data Conversion Table

Data Provided	Symbol	Model	
		Bivariate and Repairman	M/M/S
Arrival Rate	λ_i	$\lambda_{Fi} = .8(\frac{\lambda_i}{25})$ $\lambda_{Bi} = .2(\frac{\lambda_i}{25})$	$\lambda_{Fi} = .8\lambda_i$ $\lambda_{Bi} = .2\lambda_i$
Service Rate	μ_i	$\mu_{Fi} = \mu_i$ $\mu_{Bi} = \mu_i$	$\mu_{Fi} = \mu_i$ $\mu_{Bi} = \mu_i$

The division of repair personnel between flight-line and back-shop portions is not necessary in the bivariate model. But the decomposition models require that some division be made. For a shop with S_i personnel, there are $S_i - 1$ possible divisions that may be made. It was decided to make the division so that the expected number of customers in the shop was minimized. To find this division, the repairman FORTRAN program was run for each shop, with 1 through $S_i - 1$ repairmen working in the back-shop, and then run again with 1 through

$S_i - 1$ repairmen working at the flight-line. The expected number of customers for each combination of S_{Fi} and S_{Bi}, such that $S_{Fi} + S_{Bi} = S_i$, was looked at. That combination producing the minimum expected number of customers determined the division of personnel. This division of personnel was also used in the M/M/S decomposition model.

The results of the calculations for the bivariate model and the decomposition models are presented in Tables III through XII and Figures 8 through 29. In some cases, calculations for the back-shop portions of the decomposition (M/M/S) model were not made. However, there is still adequate information presented to allow a comparison of the models. The bivariate model predicts the best performance, while the decomposition (repairman) model predicts the second-best performance, and the decomposition (M/M/S) model predicts the worst performance of the three. The performance predicted by decomposition (repairman) model is in most cases very close to the performance precicted by the bivariate model, with the exception of queue lengths and delay times. The bivariate model shows the flexibility of the queueing systems presently employed by the USAF, where shop personnel can be assigned to the flight-line or the back-shop as they are needed. Queueing will not occur until all S_i personnel are busy in the bivariate model, whereas in a decomposition model, queues form when either S_{Fi} or S_{Bi} personnel are busy at the flight-line or back-shop of the system under consideration. The M/M/S queue bad performance is possibly caused by

the fact that no reduction in demand for repairmen occurs as the number in the system increases.

TABLE III

Measures of Congestion for Shop 1

Calculated from Three Different Models

	Calculations for $\lambda_{B1} = 0.001976$, $\mu_{B1} = 0.448$; $\lambda_{F1} = 0.00792$, $\mu_{F1} = 0.448$		
	Bivariate Model	Decomposition Models, $S_{F1} = 2$, $S_{B1} = 1$	
Measure	$S_1 = 3$	Repairman	M/M/S
$E(N_{F1})$	0.4337	0.4527	0.4647
$Var(N_{F1})$	0.4299	0.4818	N.C.
$E(Q_{F1})$	0.001315	0.01870	0.0227
$Var(Q_{F1})$	0.00176	0.02702	N.C.
$E(W_{F1})$	2.2391	2.3283	2.3467
$E(D_{F1})$	0.00679	0.0962	0.1146
$E(N_{B1})$	0.1098	0.1226	0.1239
$Var(N_{B1})$	0.1099	0.1361	N.C.
$E(Q_{B1})$	0.0041	0.0129	0.01367
$Var(Q_{B1})$	0.0020	0.0155	N.C.
$E(W_{B1})$	2.2711	2.4935	2.508
$E(D_{B1})$	0.0848	0.2614	0.2766

N.C. - Not calculated

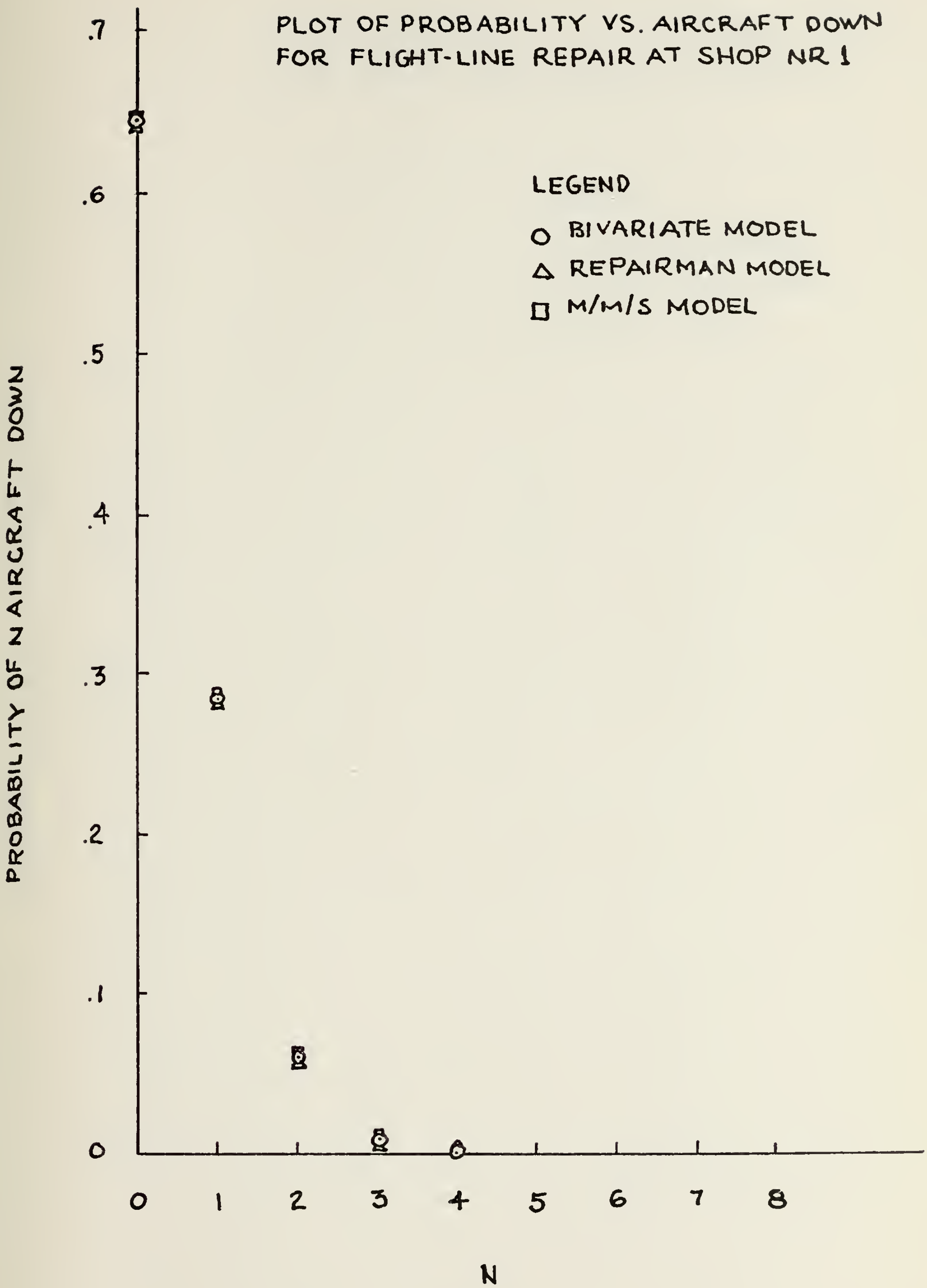

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 1
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
N

FIGURE 8

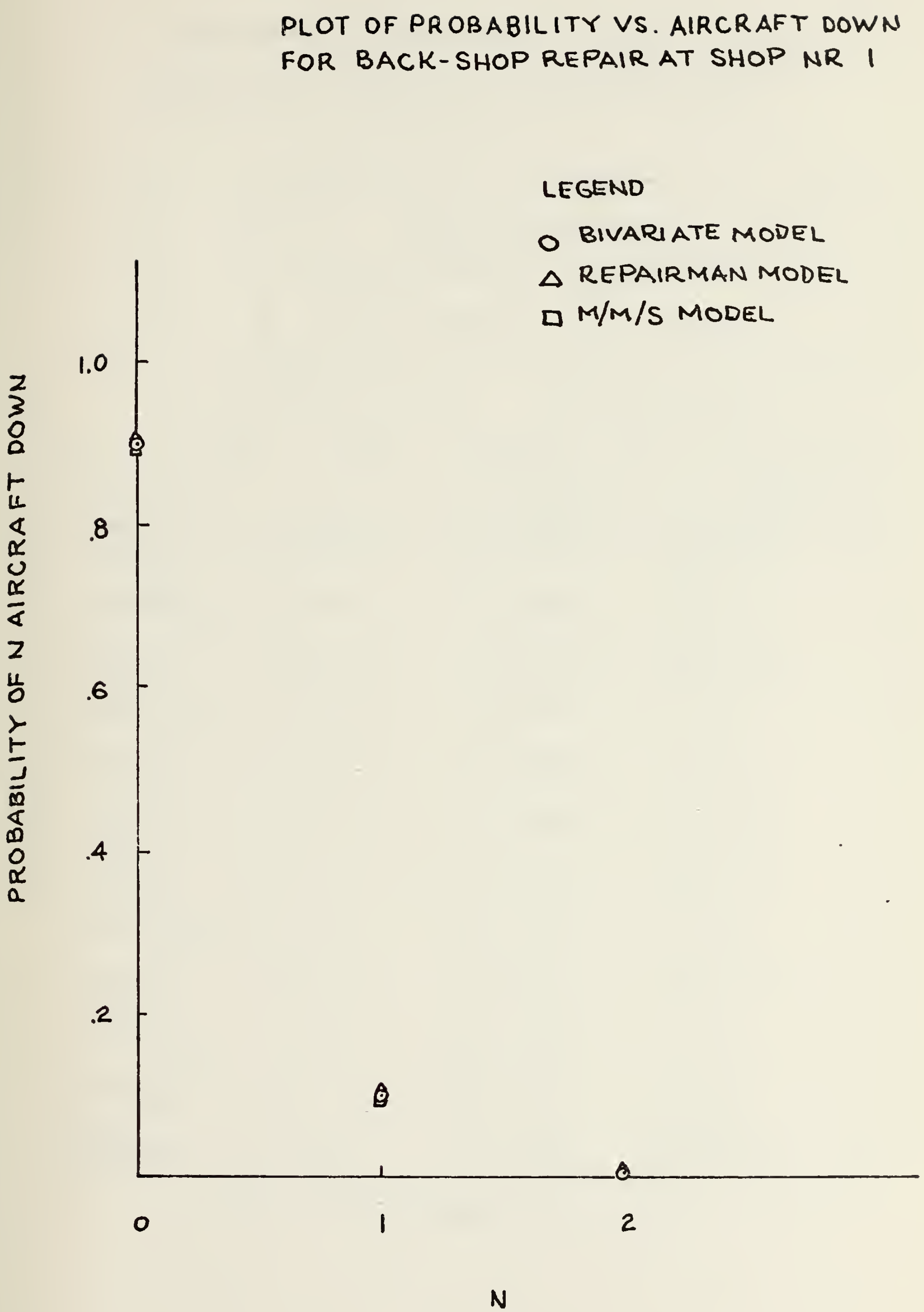

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 1
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
1.0
.8
.6
.4
.2
0
1
2
N

FIGURE 9

TABLE IV

Measures of Congestion for Shop 2

Calculated from Three Different Models

	Calculations for $\lambda_{B2} = 0.00139$, $\mu_{B2} = 0.272$; $\lambda_{F2} = 0.0056$, $\mu_{F2} = 0.272$		
	Bivariate Model	Deocomposition Models, $S_{F2} = 2$, $S_{B2} = 1$	
Measure	$S_2 = 3$	Repairman	M/M/S
$E(N_{F2})$	0.5005	0.5292	0.5467
$Var(N_{F2})$	0.4970	0.5760	N.C.
$E(Q_{F2})$	0.00237	0.0290	0.03569
$Var(Q_{F2})$	0.00310	0.0442	N.C.
$E(W_{F2})$	3.6938	3.8894	3.933
$E(D_{F2})$	0.0173	0.2129	0.2568
$E(N_{B2})$	0.1280	0.1488	0.1467
$Var(N_{B2})$	0.1286	0.1636	N.C.
$E(Q_{B2})$	0.0115	0.0176	0.0188
$Var(Q_{B2})$	0.0118	0.0219	N.C.
$E(W_{B2})$	3.773	4.184	4.216
$E(D_{B2})$	0.339	0.5081	0.5394

N.C. - Not calculated

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 2

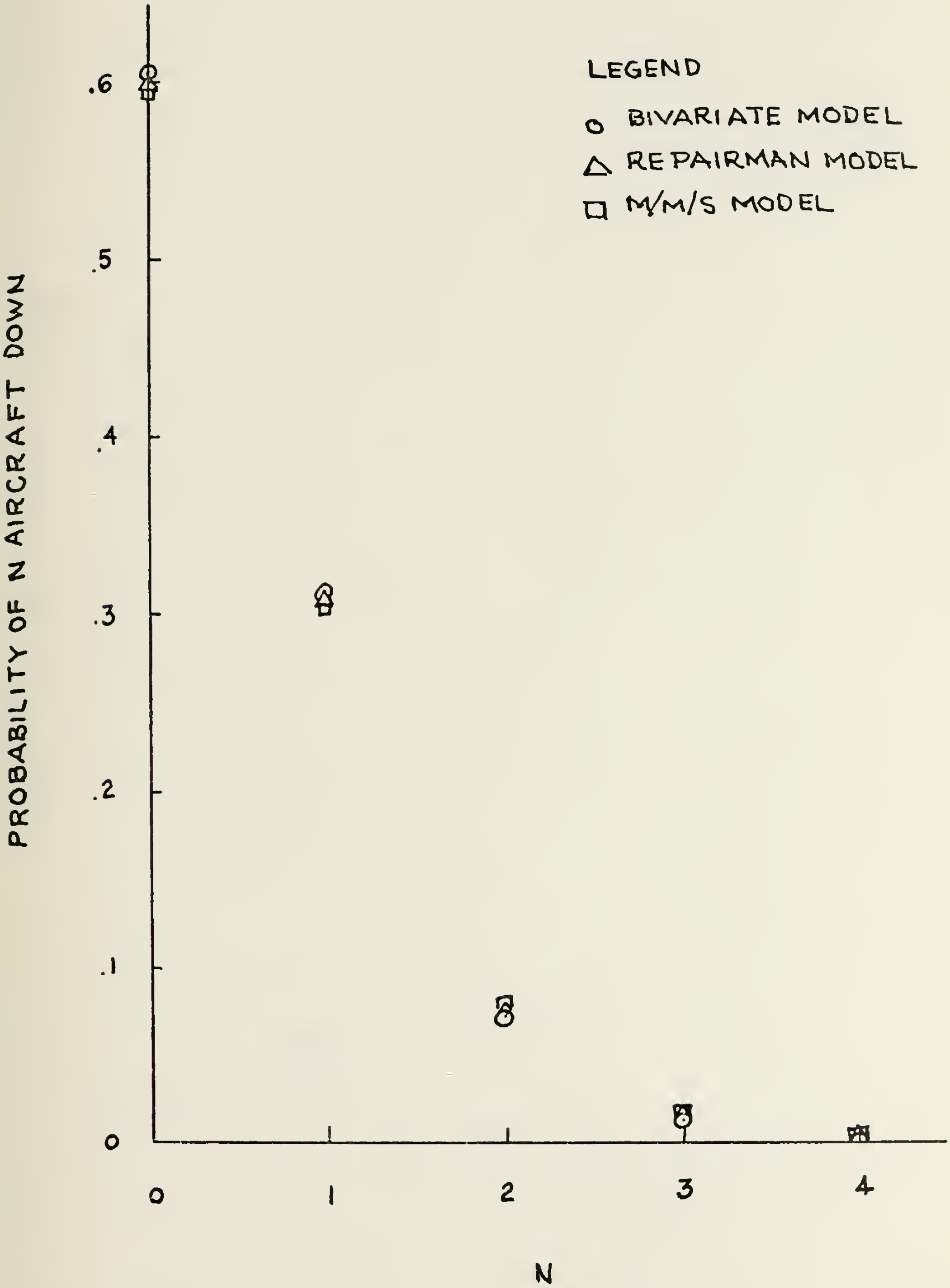


FIGURE 10

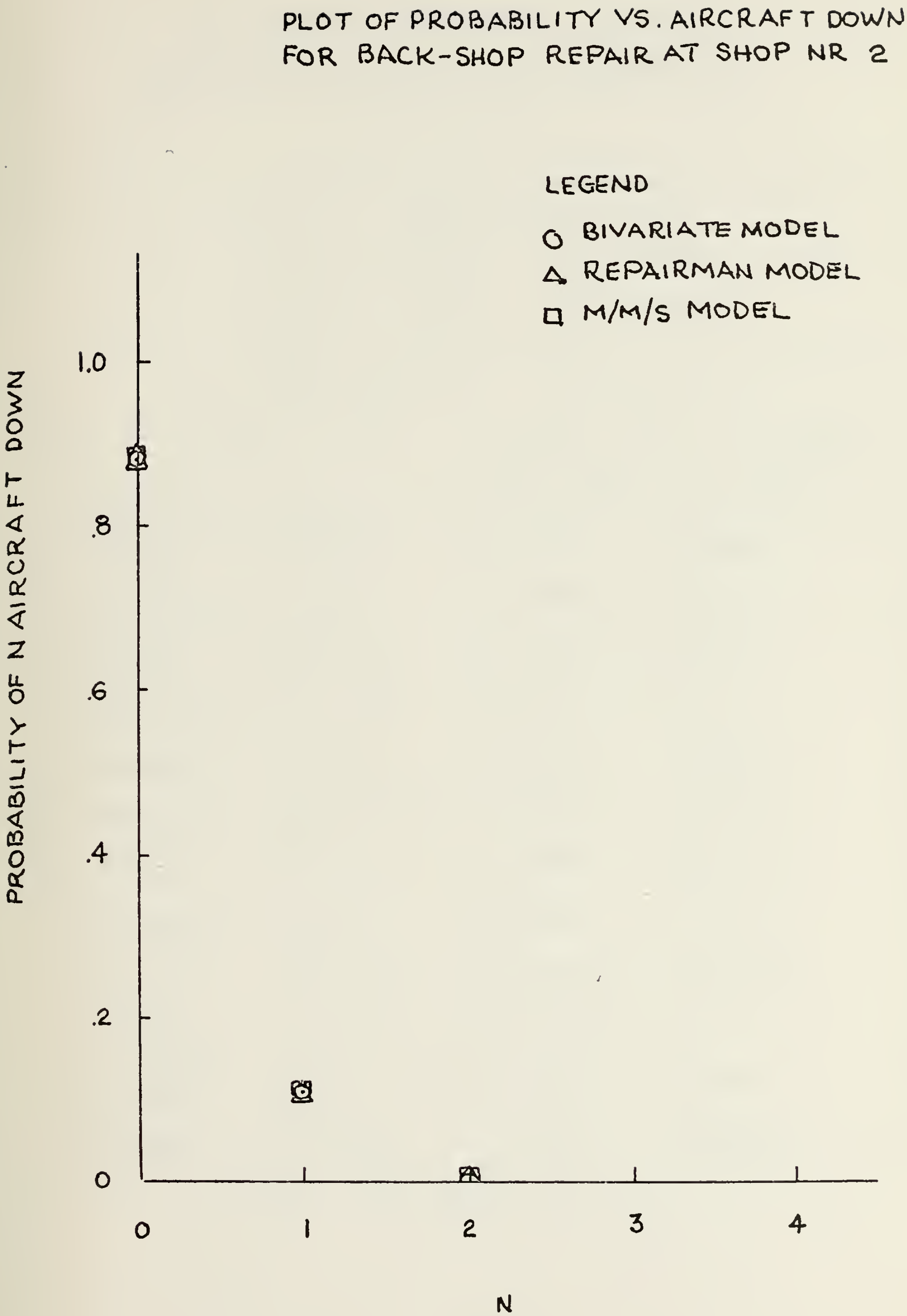

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 2
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
1.0
.8
.6
.4
.2
0
0
1
2
3
4
N

FIGURE 11

TABLE V

Measures of Congestion for Shop 3

Calculated from Three Different Models

Measure	Calculations for $\lambda_{B3} = 0.0123$, $\mu_{B3} = 0.379$, $\lambda_{F3} = 0.0492$, $\mu_{F3} = 0.379$		
	Bivariate Model $S_3 = 11$	Decomposition Models, $S_{F3} = 7$, $S_{B3} = 4$	
		Repairman	M/M/S
$E(N_{F3})$	2.7921	2.8811	3.292
$Var(N_{F3})$	2.4804	2.5926	N.C.
$E(Q_{F3})$	0.000006	0.0097	0.0469
$Var(Q_{F3})$	0.000008	0.0176	N.C.
$E(W_{F3})$	2.6385	2.6475	2.6767
$E(D_{F3})$	0.0000059	0.0089	0.0381
$E(N_{B3})$	0.6992	0.7886	0.8152
$Var(N_{B3})$	0.6797	0.7691	N.C.
$E(Q_{B3})$	0.000055	0.00154	0.0026
$Var(Q_{B3})$	0.000067	0.00211	N.C.
$E(W_{B3})$	2.639	2.644	2.647
$E(D_{B3})$	0.00021	0.0052	0.0084

N.C. - Not calculated

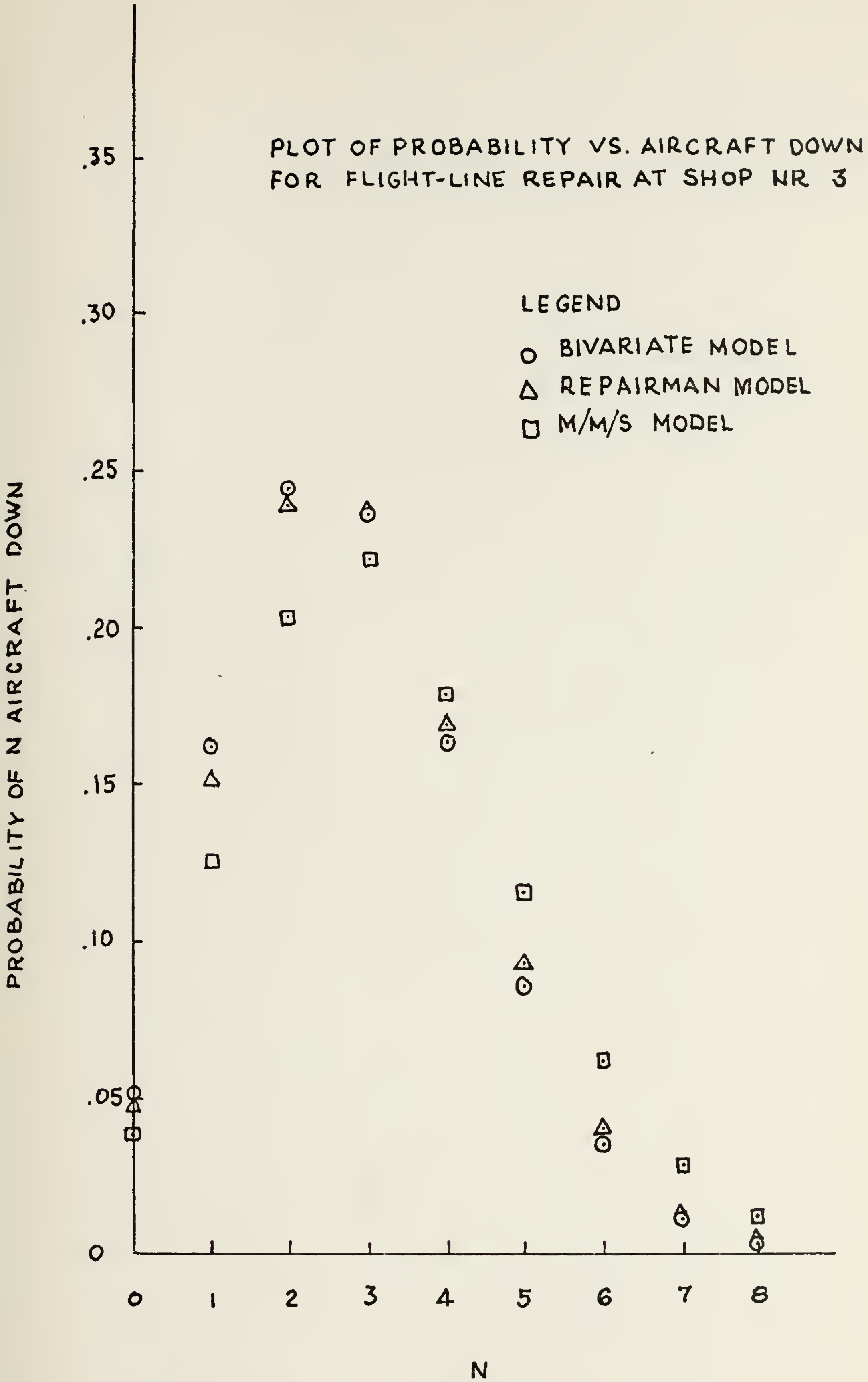

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 3
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.35
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
N

FIGURE 12

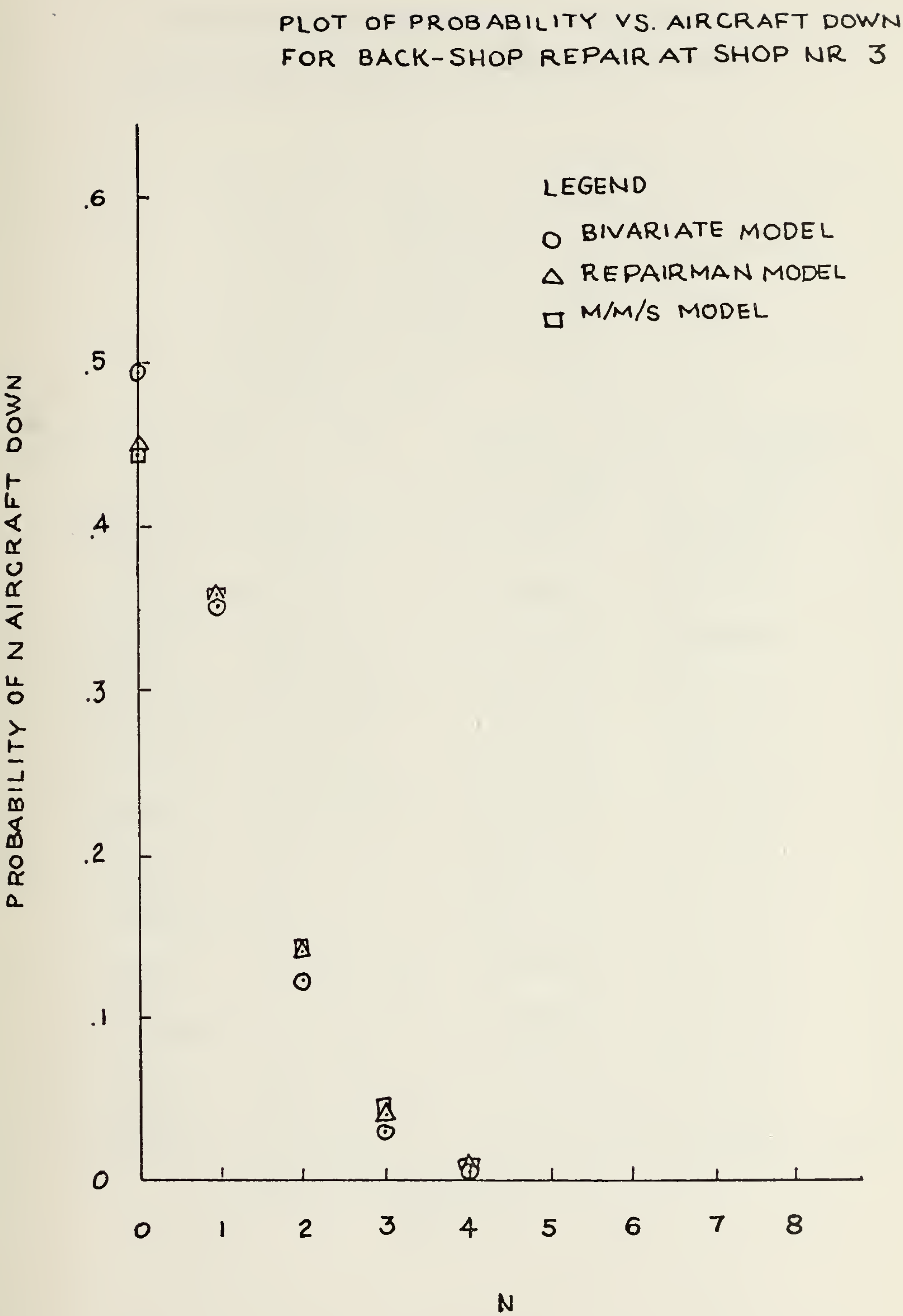

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 3
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
N

FIGURE 13

TABLE VI

Measures of Congestion for Shop 4

Calculated from Three Different Models

	Calculations for $\lambda_{B4} = 0.00732$, $\mu_{B4} = 0.313$ $\lambda_{F4} = 0.02936$, $\mu_{F4} = 0.313$		
	Bivariate Model	Decomposition Models, $S_{F4} = 6$, $S_{B4} = 3$	
Measure	$S_4 = 9$	Repairman	M/M/S
$E(N_{F4})$	2.0991	2.1505	2.3683
$Var(N_{F4})$	1.9320	1.9962	N.C.
$E(Q_{F4})$	0.000023	0.0072	0.02323
$Var(Q_{F4})$	0.000031	0.0123	N.C.
$E(W_{F4})$	3.1949	3.2056	3.227
$E(D_{F4})$	0.000035	0.0107	0.0316
$E(N_{B4})$	0.5235	0.5752	N.C.
$Var(N_{B4})$	0.5126	0.5731	N.C.
$E(Q_{B4})$	0.000128	0.0040	N.C.
$Var(Q_{B4})$	0.000179	0.0055	N.C.
$E(W_{B4})$	3.1958	3.2174	N.C.
$E(D_{B4})$	0.00078	0.0225	N.C.

N.C. - Not calculated

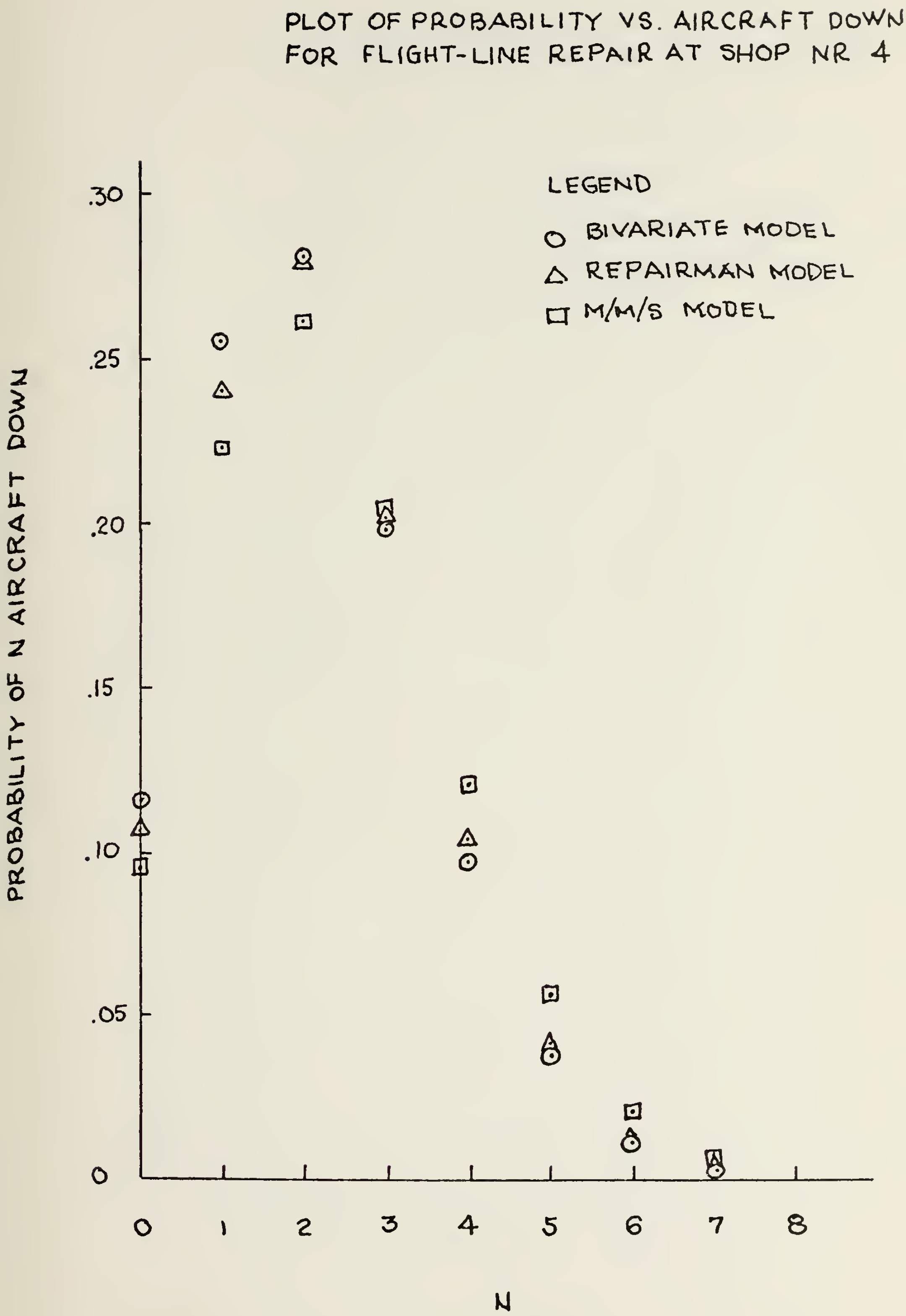

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 4
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
N

FIGURE 14

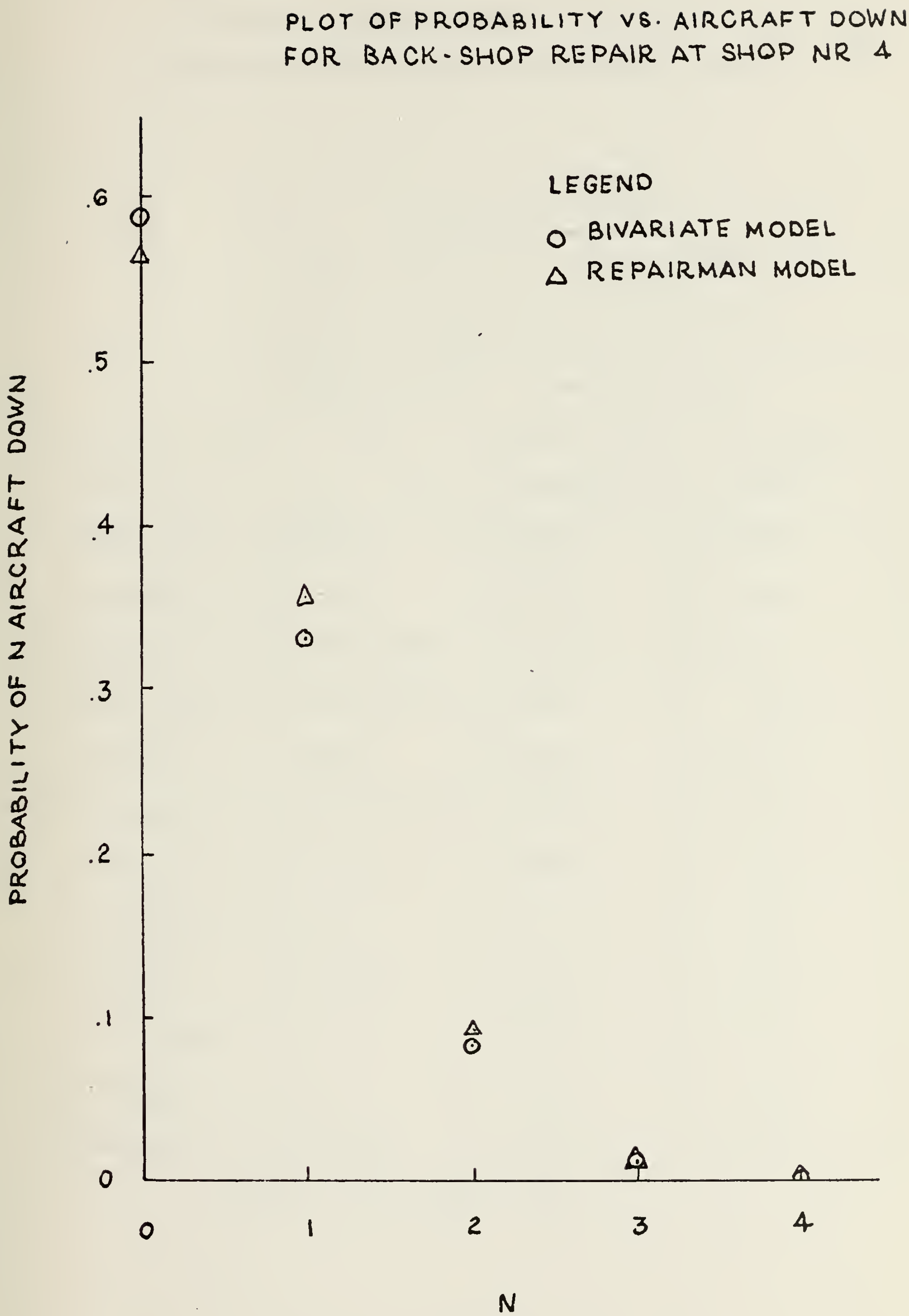

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 4
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
PROBABILITY OF N AIRCRAFT DOWN
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
N

FIGURE 15

TABLE VII

Measures of Congestion for Shop 5

Calculated from Three Different Models

	Calculations for $\lambda_{B5} = 0.01224$, $\mu_{B5} = 0.373$; $\lambda_{F5} = 0.0488$, $\mu_{F5} = 0.373$		
	Bivariate Model	Decomposition Models, $S_{F5} = 8$, $S_{B5} = 4$	
Measure	$S_5 = 12$	Repairman	M/M/S
$E(N_{F5})$	2.8213	2.8943	3.4362
$Var(N_{F5})$	2.5029	2.5703	N.C.
$E(Q_{F5})$	0.00000084	0.00214	0.1655
$Var(Q_{F5})$	0.00000107	0.0035	N.C.
$E(W_{F5})$	2.6810	2.6830	2.8166
$E(D_{F5})$	0.00000079	0.0020	0.1356
$E(N_{B5})$	0.70245	0.7959	N.C.
$Var(N_{B5})$	0.6827	0.7761	N.C.
$E(Q_{B5})$	0.00269	0.0016	N.C.
$Var(Q_{B5})$	0.0524	0.0022	N.C.
$E(W_{B5})$	2.6811	2.6864	N.C.
$E(D_{B5})$	0.01039	0.00541	N.C.

N.C. - Not calculated

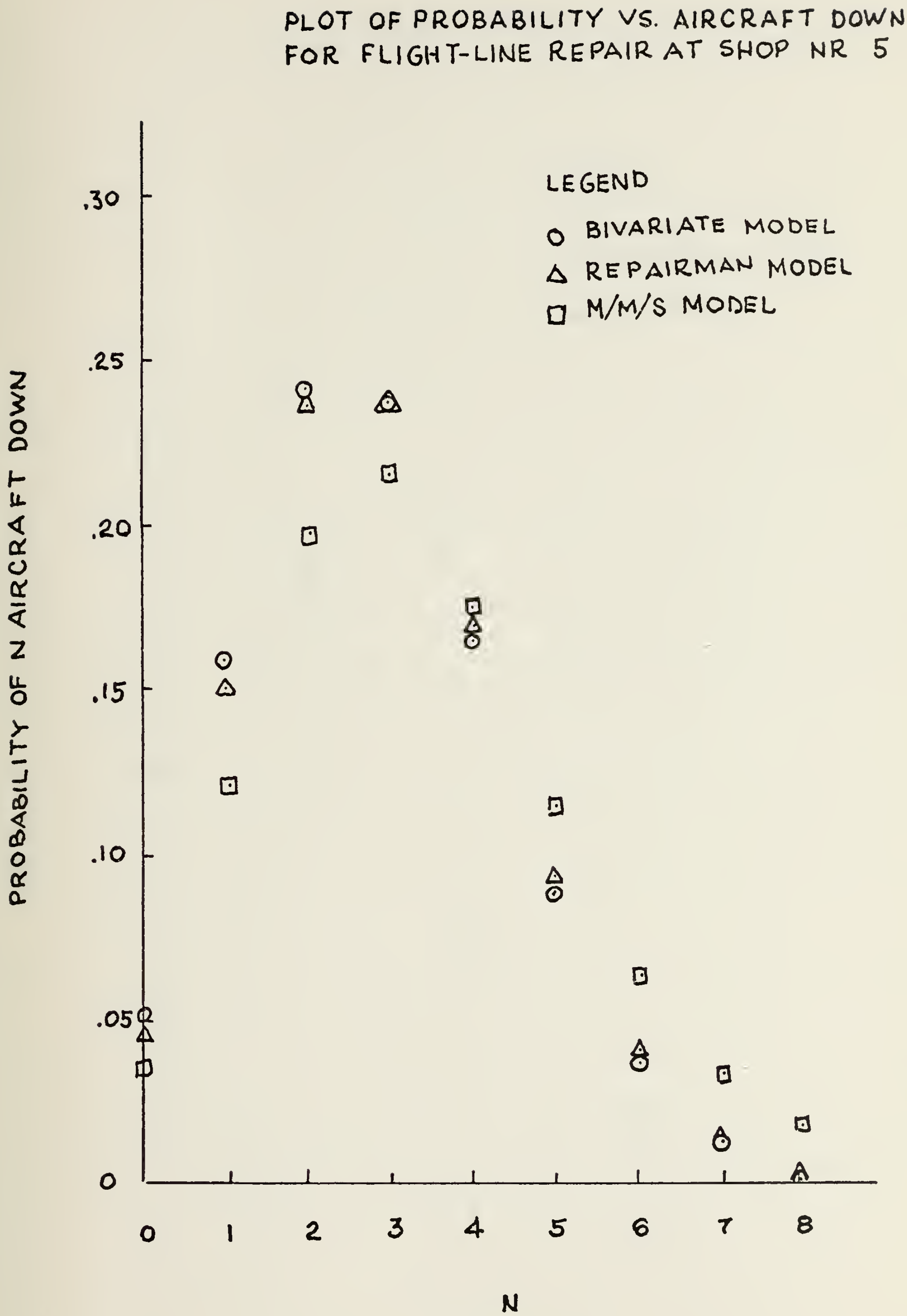

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 5
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
N

FIGURE 16

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 5

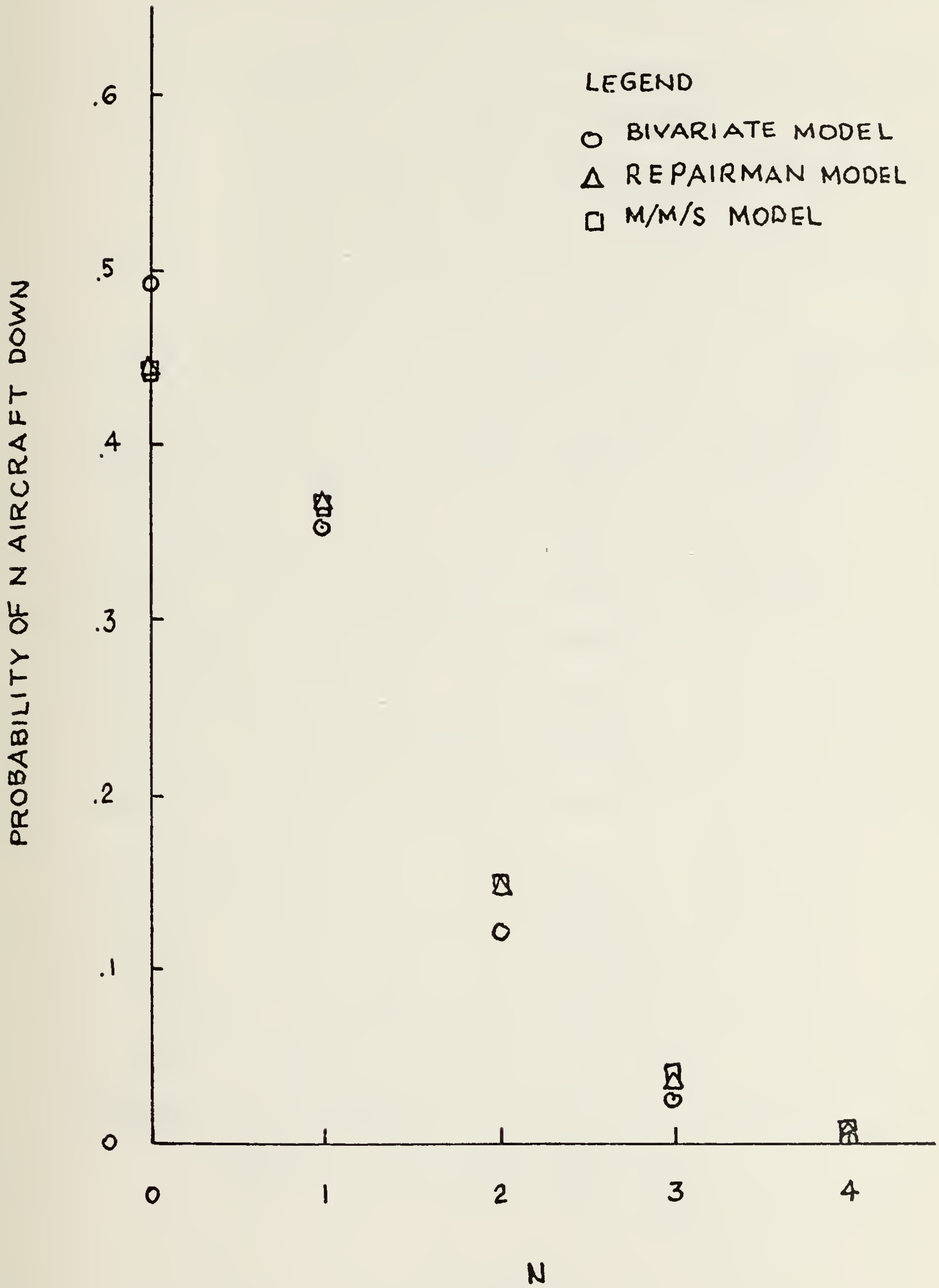


FIGURE 17

TABLE VIII

Measures of Congestion for Shop 6

Calculated from Three Different Models

Measure	Calculations for $\lambda_{B6} = 0.02256$, $\mu_{B6} = 0.775$, $\lambda_{F6} = 0.0904$, $\mu_{F6} = 0.775$		
	Bivariate Model $S_6 = 10$	Decomposition Models, $S_{F6} = 7$, $S_{B6} = 3$	
		Repairman	M/M/S
$E(N_{F6})$	2.545	2.616	3.000
$Var(N_{F6})$	2.286	2.3661	N.C.
$E(Q_{F6})$	0.00001843	0.0051	0.0839
$Var(Q_{F6})$	0.000025	0.00875	N.C.
$E(W_{F6})$	1.290	1.2928	1.3274
$E(D_{F6})$	0.00000935	0.0025	0.0371
$E(N_{B6})$	0.635	0.7161	N.C.
$Var(N_{B6})$	0.619	0.7207	N.C.
$E(Q_{B6})$	0.000138	0.00922	N.C.
$Var(Q_{B6})$	0.000182	0.01365	N.C.
$E(W_{B6})$	1.2903	1.3072	N.C.
$E(D_{B6})$	0.00028	0.0168	N.C.

N.C. - Not calculated

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 6

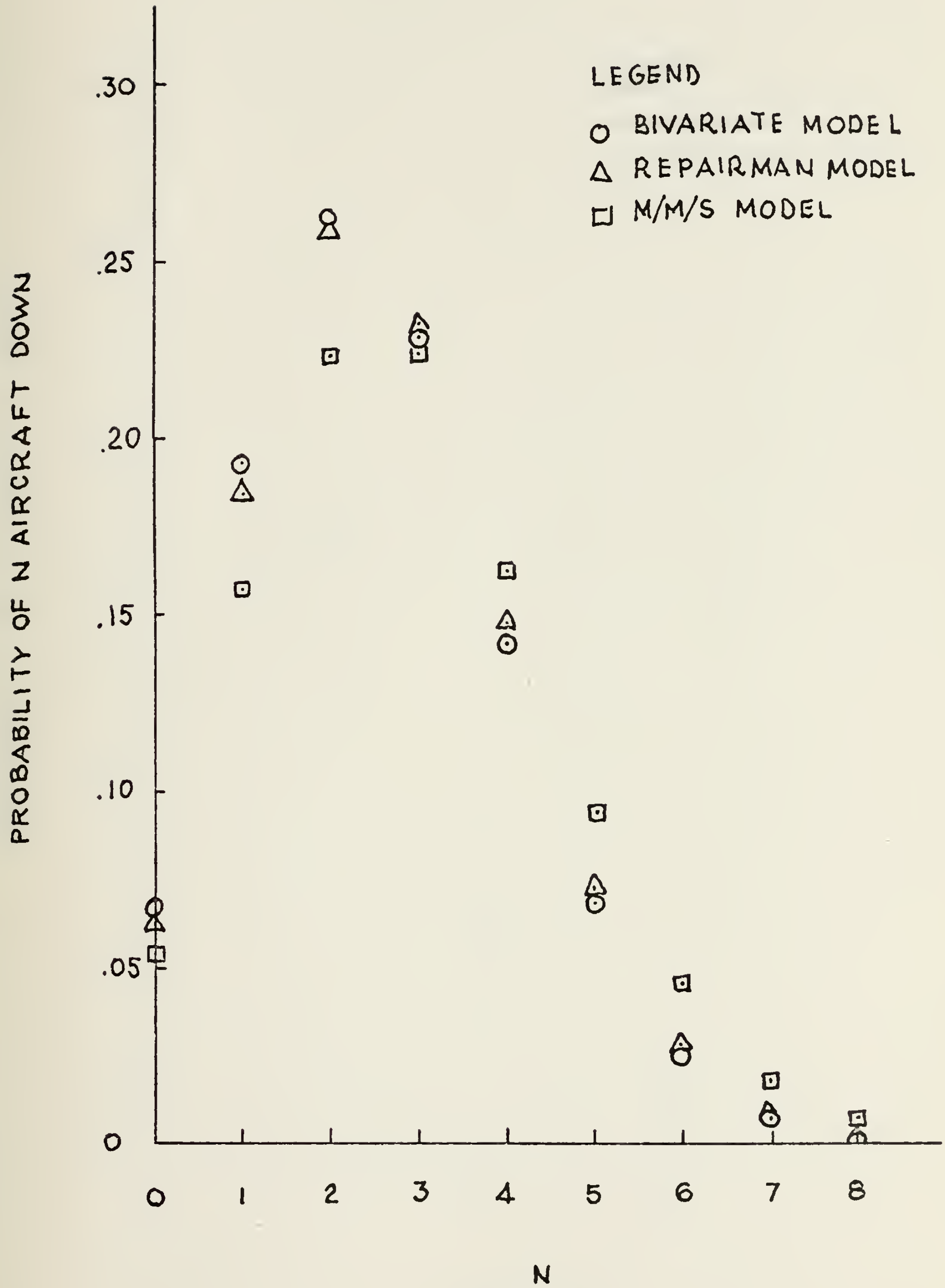


FIGURE 18

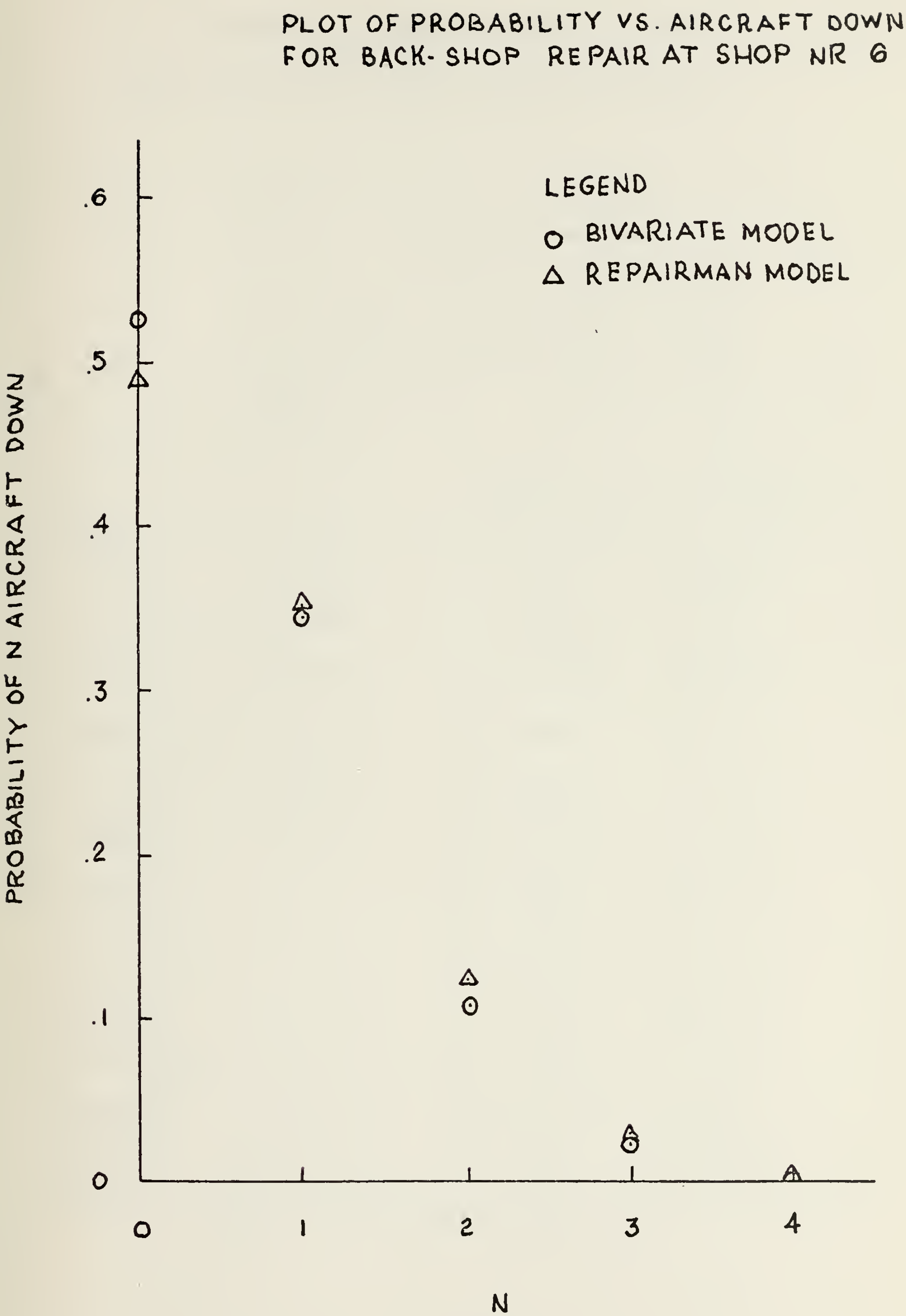

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 6
LEGEND
O BIVARIATE MODEL
Δ REPAIRMAN MODEL
PROBABILITY OF N AIRCRAFT DOWN
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
N

FIGURE 19

TABLE IX

Measures of Congestion for Shop 7

Calculated from Three Different Models

	Calculations for $\lambda_{B7} = 0.0092$, $\mu_{B7} = 0.298$; $\lambda_{F7} = 0.0368$, $\mu_{F7} = 0.298$		
	Bivariate Model	Decomposition Models, $S_{F7} = 4$, $S_{B7} = 2$	
Measure	$S_7 = 6$	Repairman	M/M/S
$E(N_{F7})$	2.6894	3.1859	4.936
$Var(N_{F7})$	2.4881	4.1567	N.C.
$E(Q_{F7})$	0.0257	0.49206	1.8490
$Var(Q_{F7})$	0.0493	1.3335	N.C.
$E(W_{F7})$	3.3880	3.9687	5.3654
$E(D_{F7})$	0.0323	0.6130	2.0097
$E(N_{B7})$	0.7401	0.8476	0.9069
$Var(N_{B7})$	0.7725	1.0286	N.C.
$E(Q_{B7})$	0.0756	0.1019	0.1351
$Var(Q_{B7})$	0.1335	0.1869	N.C.
$E(W_{B7})$	3.7297	3.8145	3.943
$E(D_{B7})$	0.3810	0.4587	0.5872

N.C. - Not calculated

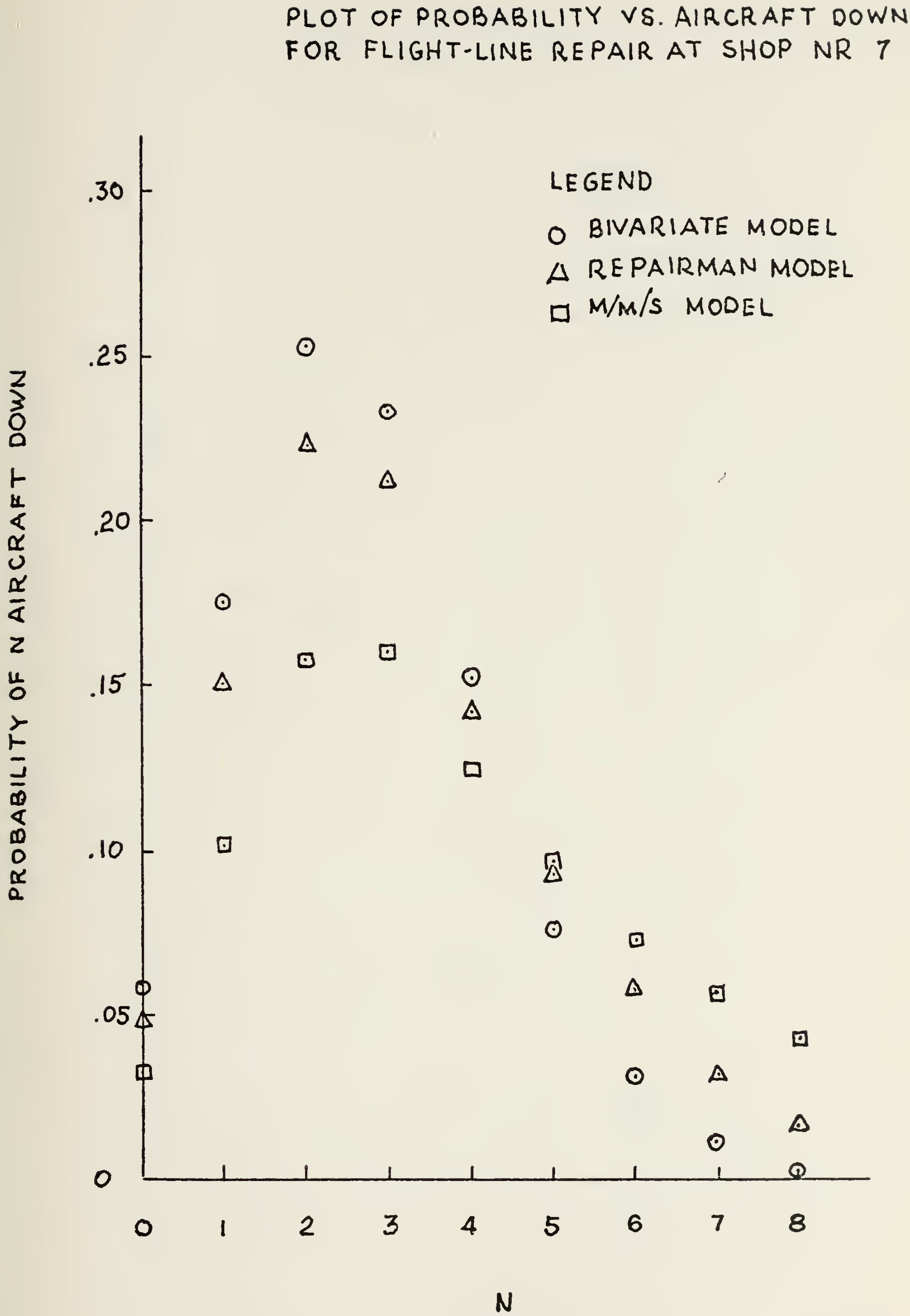

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 7
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
N

FIGURE 20

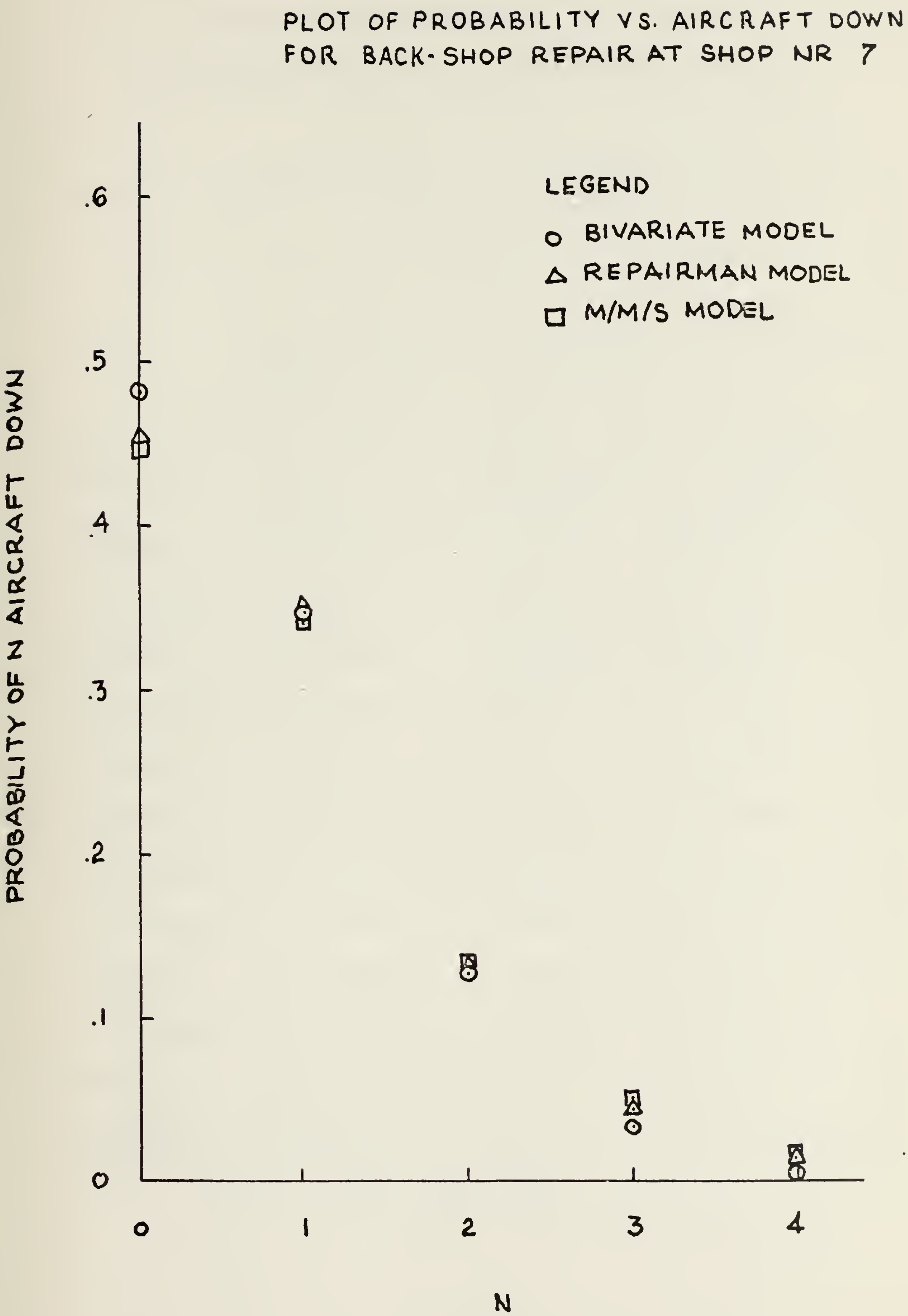

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 7
LEGEND
o BIVARIATE MODEL
△ REPAIRMAN MODEL
□ M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
N

FIGURE 21

TABLE X

Measures of Congestion for Shop 8

Calculated from Three Different Models

	Calculations for $\lambda_{B8} = 0.00572$, $\mu_{B8} = 0.472$; $\lambda_{F8} = 0.0228$, $\mu_{F8} = 0.472$		
	Bivariate Model	Decomposition Models, $S_{F8} = 5$, $S_{B8} = 2$	
Measure	$S_8 = 7$	Repairman	M/M/S
$E(N_{F8})$	1.1426	1.1570	1.2146
$Var(N_{F8})$	1.0904	1.1087	N.C.
$E(Q_{F8})$	0.000013	0.00126	0.0027
$Var(Q_{F8})$	0.000017	0.0018	N.C.
$E(W_{F8})$	2.119	2.1210	2.1234
$E(D_{F8})$	0.000025	0.0023	0.0048
$E(N_{B8})$	0.2857	0.3053	N.C.
$Var(N_{B8})$	0.2825	0.3135	N.C.
$E(Q_{B8})$	0.000051	0.0060	N.C.
$Var(Q_{B8})$	0.000069	0.0078	N.C.
$E(W_{B8})$	2.1192	2.1613	N.C.
$E(D_{B8})$	0.000378	0.0426	N.C.

N.C. - Not calculated

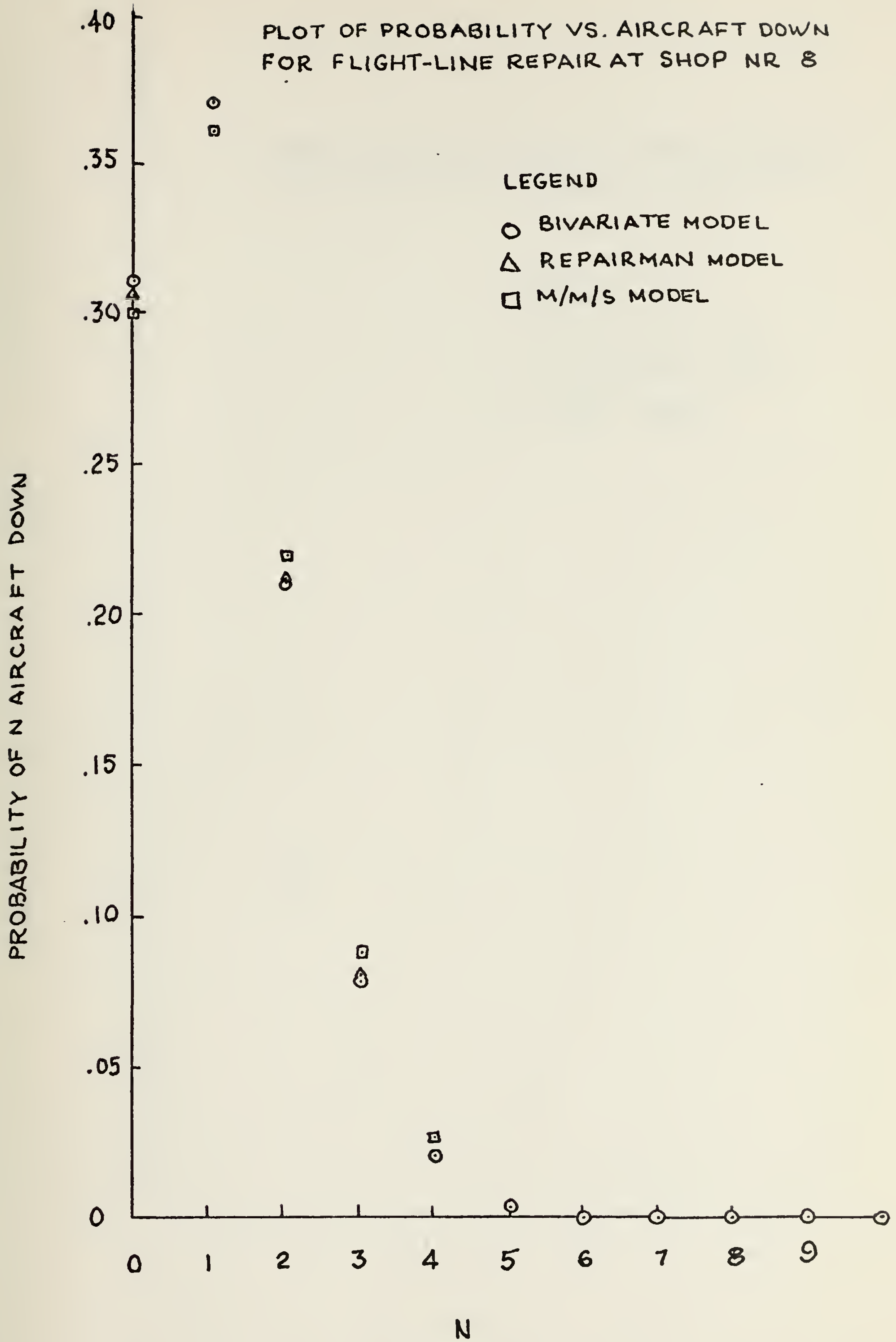

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 8
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.40
.35
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
9
N

FIGURE 22

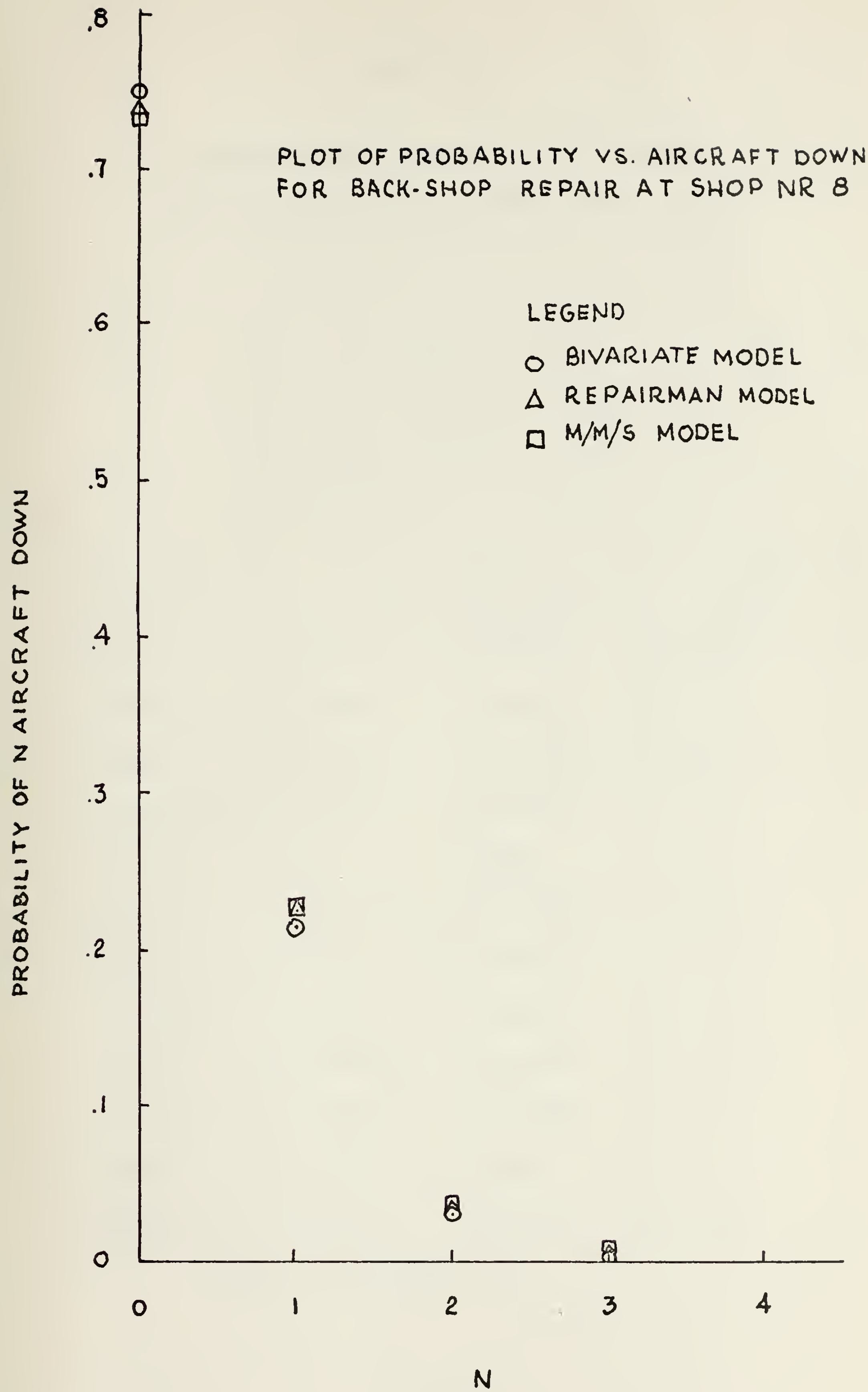

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 8
LEGEND
BIVARIATE MODEL
REPAIRMAN MODEL
M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.8
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
N

FIGURE 23

TABLE XI

Measures of Congestion for Shop 9

Calculated from Three Different Models

Measure	Calculations for $\lambda_{B9} = 0.02216$, $\mu_{B9} = 0.333$, $\lambda_{F9} = 0.0884$, $\mu_{F9} = 0.333$		
	Bivariate Model $S_9 = 23$	Decomposition Models, $S_{F9} = 15$, $S_{B9} = 8$	
		Repairman	M/M/S
$E(N_{F9})$	4.9324	5.2444	6.6397
$Var(N_{F9})$	3.9394	4.1442	N.C.
$E(Q_{F9})$	0.000000	0.000000	0.0031
$Var(Q_{F9})$	0.000000	0.00001	N.C.
$E(W_{F9})$	3.003	3.003	3.004
$E(D_{F9})$	0.000000	0.000000	0.0014
$E(N_{B9})$	1.2490	1.5599	N.C.
$Var(N_{B9})$	1.1866	1.4626	N.C.
$E(Q_{B9})$	0.000000	0.00002	N.C.
$Var(Q_{B9})$	0.000000	0.00002	N.C.
$E(W_{B9})$	3.0030	3.003	N.C.
$E(D_{B9})$	0.000000	0.00003	N.C.

N.C.- Not calculated

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 9

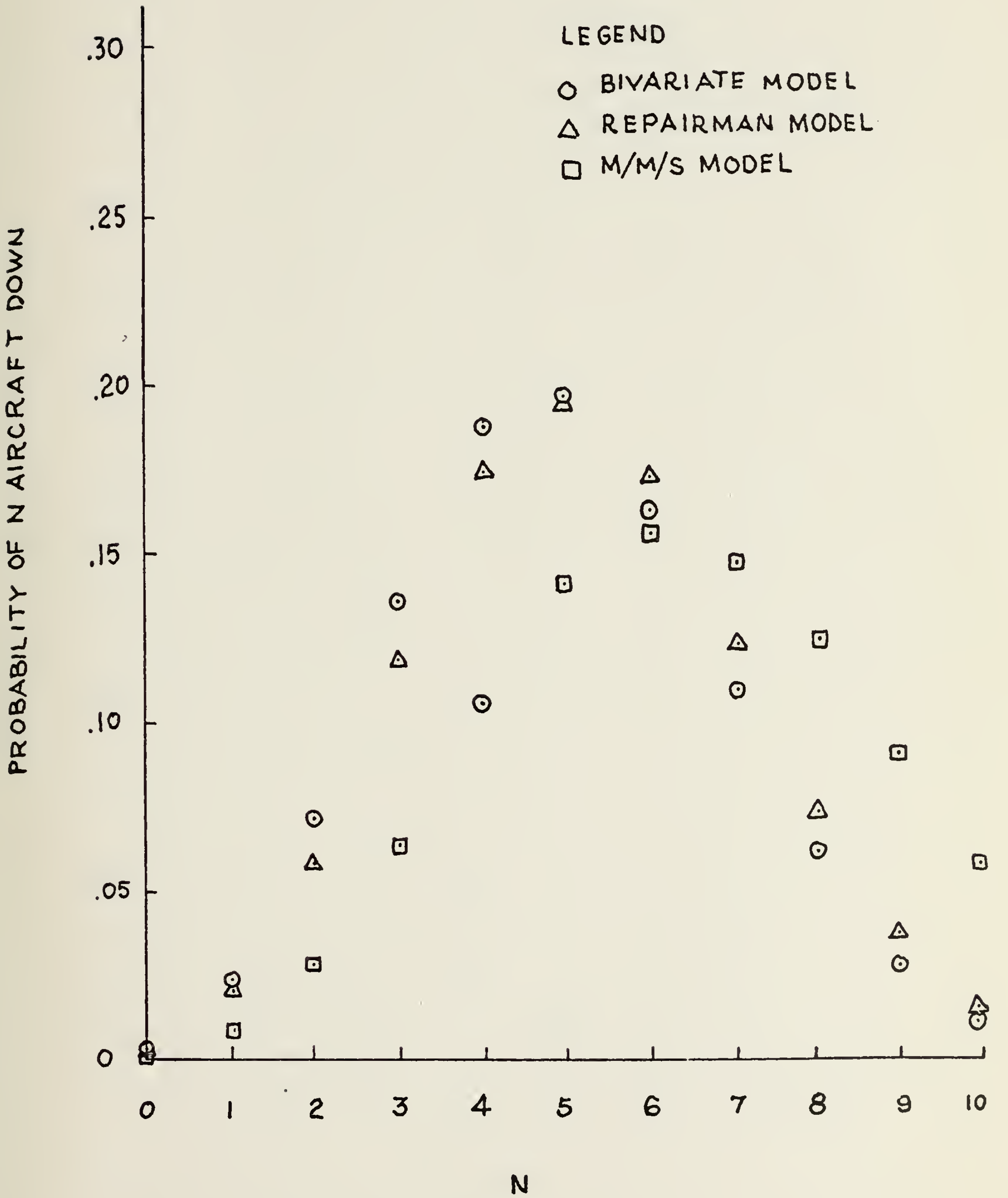


FIGURE 24

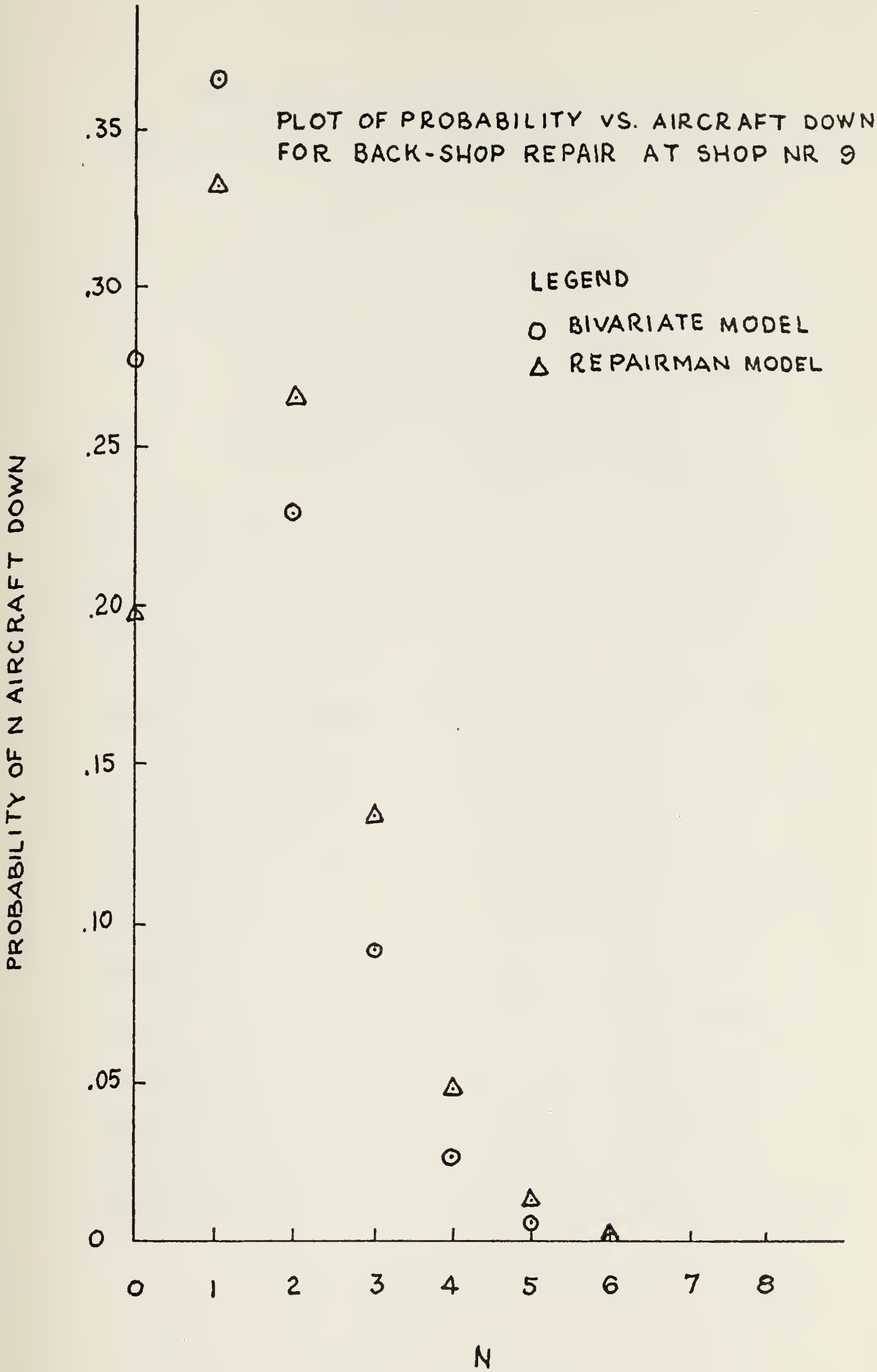

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 9
LEGEND
O BIVARIATE MODEL
Δ REPAIRMAN MODEL
PROBABILITY OF N AIRCRAFT DOWN
.35
.30
.25
.20
.15
.10
.05
0
0
1
2
3
4
5
6
7
8
N

FIGURE 25

TABLE XII

Measures of Congestion for Shop 10

Calculated from Three Different Models

Measure	Calculations for $\lambda_{B10} = 0.00133$, $\mu_{B10} = 0.389$; $\lambda_{F10} = 0.00532$, $\mu_{F10} = 0.389$		
	Bivariate Model $S_{10} = 3$	Decomposition Models, $S_{F10}=2$, $S_{B10}=1$	
		Repairman	M/M/S
$E(N_{F10})$	0.3366	0.3458	0.3522
$Var(N_{F10})$	0.3335	0.3582	N.C.
$E(Q_{F10})$	0.00005	0.00865	0.01029
$Var(Q_{F10})$	0.00006	0.01155	N.C.
$E(W_{F10})$	2.5745	2.6366	2.648
$E(D_{F10})$	0.0038	0.0659	0.0774
$E(N_{B10})$	0.0846	0.0926	0.0896
$Var(N_{B10})$	0.0845	0.1003	N.C.
$E(Q_{B10})$	0.000712	0.00753	0.00737
$Var(Q_{B10})$	0.000807	0.00874	N.C.
$E(W_{B10})$	2.592	2.7982	2.801
$E(D_{B10})$	0.0218	0.22759	0.2304

N.C. - Not calculated

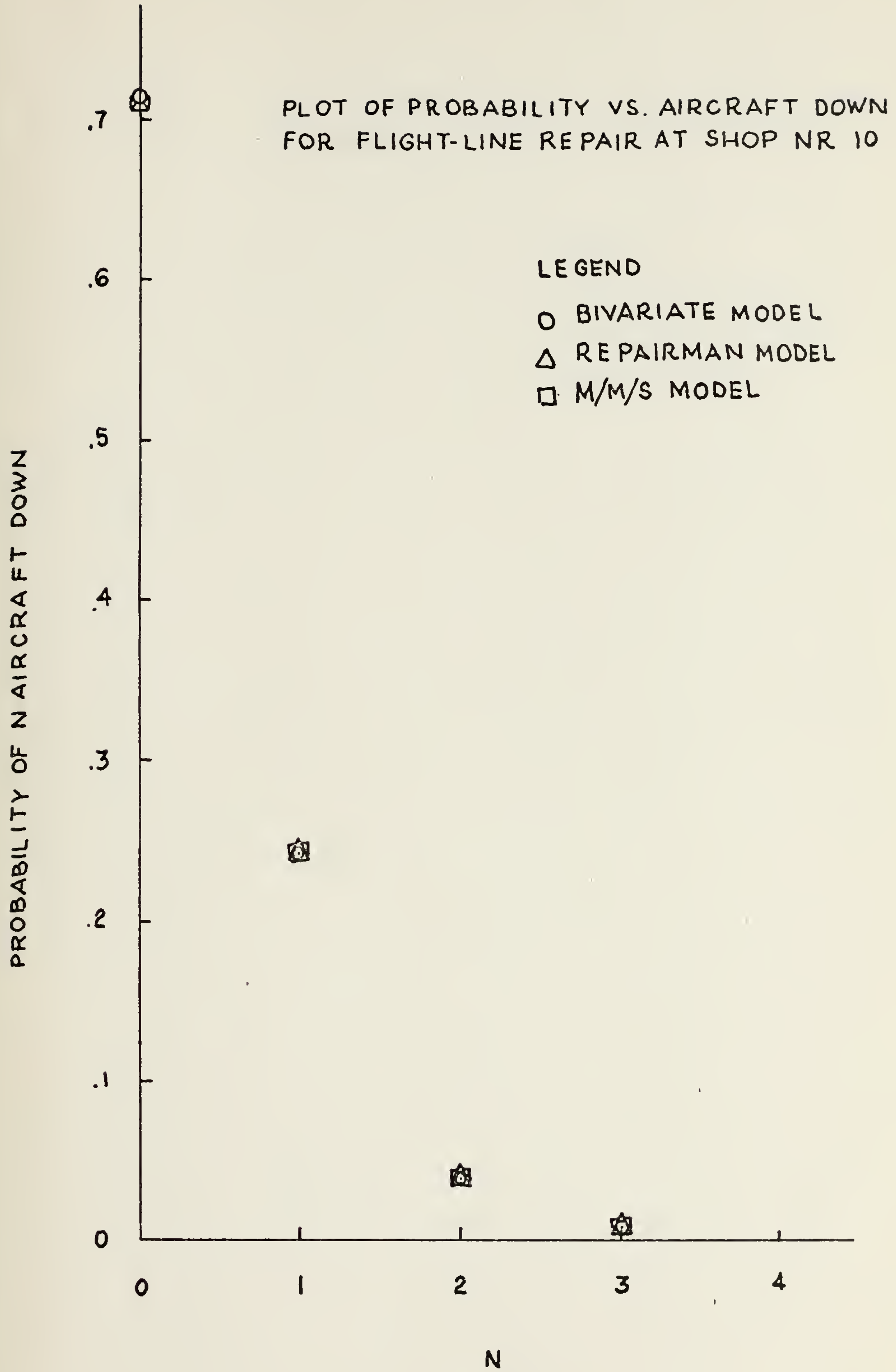

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR FLIGHT-LINE REPAIR AT SHOP NR 10
LEGEND
O BIVARIATE MODEL
Δ REPAIRMAN MODEL
□ M/M/S MODEL
PROBABILITY OF N AIRCRAFT DOWN
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
N

FIGURE 26

PLOT OF PROBABILITY VS. AIRCRAFT DOWN
FOR BACK-SHOP REPAIR AT SHOP NR 10

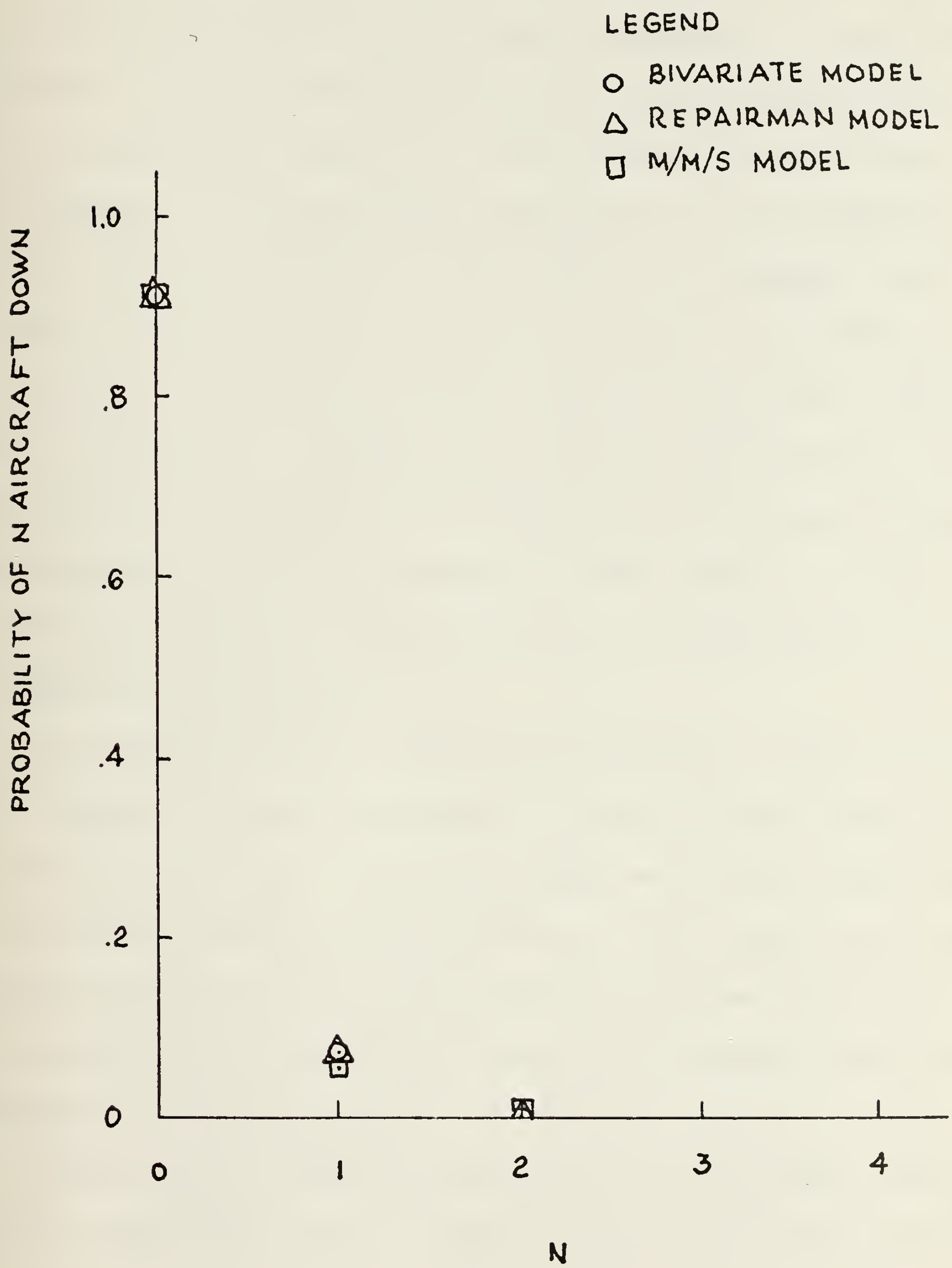


FIGURE 27

VI. CALCULATION OF MANPOWER SAVINGS

When wilt thou save the people?
  Oh, God of Mercy! when?
The people, Lord, the people!
  Not thrones and crowns, but men!
                -Ebenezer Elliot, Poetical Works

One application of the models developed in the previous chapters is the determination of manpower savings attainable from a scaled-up facility. Specifically, the savings achievable if the back-shop functions of five identical bases were consolidated into a large back-shop facility was determined. Each base was assumed to consist of 10 shops, each identical to Shops 1 through 10 used in the model comparisons made in Chapter V. Each base operates 25 aircraft with failure characteristics to those used in the model comparisons. The scheme was to leave crews at the 5 bases to handle only flight-line repairs. Back-shop repairs were made at the consolidated back-shop.

The decomposition (repairman) model was used to determine the number of repair personnel to leave at each base for flight-line duties. The back-shops were manned from the pool of remaining maintenance personnel. The consolidated back-shops were modeled as M/M/S queues since the number of aircraft failing was large enough (125) so that the model predicts reasonably well.

Each base was manned with a total of 87 repair personnel at the ten shops, with 58 assigned to flight-line duties and

29 assigned to back-shop duties (according to the repairman model). For 5 bases there were 435 total men available for the base consolidation, with 290 assigned to flight-line duties, and a pool of 145 men available for assignment to the consolidated back-shops.

It was decided to assign men to the consolidated back-shops so that the expected number of aircraft down at each shop was within 10% of the number predicted by the decomposition (repairman) model, with shops manned as in Chapter V. For example, if the expected number of aircraft in the back-shop of Shop i was 1.00 before the consolidation, then 5 bases would have, on the average, 5.00 men in the back-shops of Shop i. The standard for the consolidated back-shop would then be an expected number of aircraft down of less than 5.5.

The arrival rate used at the consolidated back-shop of Shop i was $125\lambda_{Bi}$. The service rates were unchanged. The M/M/S queue model was run for various manning levels, until the expected number of aircraft in the shop dropped below the standard discussed above. Table XIII presents the result of the calculations of four measures of congestion made for the consolidated back-shop facility, and also shows the manpower savings achieved.

A total of 89 personnel were eliminated by consolidation of back-shop repairs. This is roughly the former manning level of one base, or a savings of roughly 20% achieved by scaling-up the back-shop functions. Recall, that the total number of shops at the base from which the data was collected

TABLE XIII

Manpower Savings and Measures of Congestion
for a Consolidated Back-Shop Facility
(Five Bases Consolidated)

Shop Nr. (i)	Men Available	Men Assigned	Savings	$E(N_{Bi})$	$E(Q_{Bi})$	$E(W_{Bi})$	$E(D_{Bi})$
1	5	2	3	0.597	0.045	2.416	0.1835
2	5	2	3	0.713	0.0079	3.722	0.0455
3	20	7	13	4.129	0.1995	3.760	0.1295
4	15	6	9	3.008	0.0851	3.288	0.0930
5	20	7	13	4.314	0.21215	2.819	0.1387
6	15	7	8	3.737	0.0978	1.325	0.0347
7	10	6	4	4.312	0.4534	3.750	0.3942
8	10	4	6	1.5617	0.0469	2.184	0.0655
9	40	13	27	8.4915	0.1732	3.065	0.060
10	5	2	3	0.447	0.02035	2.693	0.1226

was twenty. The number of men saved by reorganization should be roughly twice that predicted here, or a little less than 200 repair personnel.

VII. CONCLUSIONS

'Is there any point to which you wish to draw my attention?'
'To the curious incident of the dog in the night-time.'
'The dog did nothing in the night-time.'
'That was the curious incident,' remarked Sherlock Holmes.
-Sir Arthur Conan Doyle, The Memoirs of Sherlock Holmes

The decomposition (repairman) model was in close agreement with the bivariate model, except with respect to queue lengths and delay times. It is quite suitable for use in problems such as base consolidation studies, where savings of repair personnel are sought. The decomposition (M/M/S) model provides conservative predictions, which makes it more suitable for studies made for a risk-averse decision maker.

If performance predictions are desired for repair configurations similar to that described in Chapter II, where the effects of sustained changes in tempo of operations or the effect on system performance of manpower reduction or augmentation is desired, then the bivariate model is more appropriate than either of the two decomposition models. Furthermore, although all three models are over-simplified, the balance equations describing more complex situations can easily be written down. Then they may also be solved using Gauss-Seidel iteration. Thus techniques to replace complex simulation, at least in some instances, are at hand. Such work is under continuing development.

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